13/3/22

הרצאה 1

← **אינטגרלים**

$f(x)=x^2+C$

$f'(x)= 2x$

$f(x) = x^2$

$f(x)= x^2+17$

x^2-5

$$\int 2x \; dx = x^2+C$$

$$\int x^2 dx = \frac{x^3}{3}+C$$

$f'(x) = x^2$

$f(x) = \frac{x^3}{3}+C$

$f(x)$	$\int f(x)\,dx$
x^n	$\frac{x^{n+1}}{n+1}+C$ $\quad n\neq -1$
$\frac{1}{\sqrt{x}}$	$2\sqrt{x}+C$
e^x	e^x+C
a^x	$\frac{a^x}{\ln a}+C$
$x^{-1}=\frac{1}{x}$	$\ln x+C$
$\sin x$	$-\cos x+C$
$\cos x$	$\sin x+C$
$\frac{1}{\cos^2 x}$	$\tan x$
$\frac{1}{\sin^2 x}$	$\cot x$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x$
$\frac{1}{x^2+1}$	$\arctan x$

$$\int x^3\,dx = \frac{x^4}{4} + C$$

$$\int x^8\,dx = \frac{x^9}{9} + C$$

$$\int x^{-10}\,dx = \frac{x^{-9}}{-9} + 9$$

$$\int x^{\frac{3}{7}}\,dx = \frac{x^{\frac{10}{7}}}{\frac{10}{7}} + C$$

$$\int x^{-\frac{5}{6}}\,dx = \frac{x^{\frac{1}{6}}}{\frac{1}{6}} + C$$

$$\int 1\,dx = x + C$$

$$\int 8\,dx = 8x + C$$

$$\int x^{-1}\,dx = \ln x + C$$

$$\int \left(f(x) \pm g(x)\right) dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int \left(x^5 - x^3 + x^{10}\right) dx = \frac{x^6}{6} - \frac{x^4}{4} + \frac{x^{11}}{11} + C$$

$$\int k\,f(x)\,dx = k\int f(x)\,dx$$

$$\int 8x^5\,dx = 8\int x^5\,dx = 8\cdot\frac{x^6}{6} + C$$

$$\int 3x^7\,dx = \frac{3x^8}{8} + C$$

$$\int -4x^9\,dx = \frac{-4x^{10}}{10} + C$$

$$\int 8x^{-3}\,dx = \frac{8x^{-2}}{-2} + C$$

$$\int (3x+5)\,dx = \frac{3x^2}{2} + 5x + C$$

$$\int (9x+6) = \frac{9x^2}{2} + 6x + C$$

$$\int \left(7x^4 - 5x^{-4} + 9x^{-2} + 7x^{10} - 3x^{-1} + 2x^2 - 7x - 5\right) dx$$

$$= \frac{7x^5}{5} - \frac{5x^{-3}}{-3} + \frac{9x^{-1}}{-1} + \frac{7x^{11}}{11} - 3\ln x + \frac{2x^3}{3} - \frac{7x^2}{2} - 5x + C$$

$\int f(x)\cdot g\cancel{(x)}\,dx =$ אין נוסחה כללית

$\int \frac{f(x)}{g(x)}\,\cancel{dx} =$ אין נוסחה כללית

$$\Rightarrow \int (3x+5)^{10}\, dx = \frac{(3x+5)^{11}}{11 \cdot 3} + C$$

$$\Rightarrow \int (7x+9)^{-8}\, dx = \frac{(7x+9)^{-7}}{-7 \cdot 7} + C$$

$$\Rightarrow \int (9-3x)^{14}\, dx = \frac{(9-3x)^{15}}{15 \cdot -3}$$

$ax+b$

$$\int f(x)\, dx = F(x) + C \quad \text{OK} \Leftarrow$$

$$\int f(ax+b)\, dx = \frac{1}{a} F(ax+b) + C \quad \text{'SK}$$

$$\Rightarrow \int (3x-5)^{7}\, dx = \frac{(3x-5)^{8}}{8 \cdot 3} + C$$

$$\Rightarrow \int \frac{1}{3x-5}\, dx = \frac{2\sqrt{3x-5}}{3} + C$$

$$\Rightarrow \int e^{3x-5}\, dx = \frac{e^{3x-5}}{3} + C$$

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$$\Rightarrow \int 7^{3x-5}\, dx = \frac{7^{3x-5}}{\ln 7 \cdot 3} + C$$

$$\Rightarrow \int \frac{1}{3x-5}\, dx = \frac{\ln(3x-5)}{3} + C$$

$$\Rightarrow \int \sin(3x-5)\, dx = \frac{-\cos(3x-5)}{3}$$

$$\int \sin(x^2)$$

$$\cancel{\frac{-\cos(x^2)}{2x}}$$

$$\Rightarrow \int \sqrt[3]{x^5}\,dx = \int x^{\frac{5}{3}}\,dx = \frac{x^{\frac{8}{3}}}{\frac{8}{3}} + C$$

$$\Rightarrow \int \frac{1}{\sqrt[5]{(6-2x)^4}}\,dx = \int (6-2x)^{-\frac{4}{5}}\,dx = \frac{(6-2x)^{\frac{1}{5}}}{\frac{1}{5}\cdot -2}$$

$$\Rightarrow \int (3\sqrt{x}-5)\,(2x+6x\sqrt{x})\,dx = \int (3x^{\frac{1}{2}}-5)(2x+6\cdot x^{\frac{3}{2}})\,dx$$

$$= \int (6x^{1.5}+10x^2-10x-30x^{1.5})\,dx = \frac{6x^{2.5}}{2.5} + \frac{10x^3}{3} - \frac{10x^2}{2} - \frac{30x^{2.5}}{2.5} + C$$

$$\Rightarrow \int \frac{(7x\sqrt{x}-4)\,(3x+2\sqrt{x})}{x^3\cdot\sqrt{x}}$$

$$\hookrightarrow \int \frac{(7x^{\frac{3}{2}}-4)\,(3x+2^{\frac{1}{2}})}{x^{\frac{7}{2}}} \Rightarrow \cdots\cdots = \int 21x^{-1} + 14x^{-1.5} - 12x^{-2.5} \cdots$$

$$\Rightarrow \int (\sin x + \cos x)^2\,dx = \int \left(\left(\overbrace{\sin^2 x + 2\sin x\cos x + \cos^2 x}^{1}\right)\right) dx =$$

$$= \int (1+\sin 2x)\,dx = x - \frac{\cos 2x}{2} + C$$

$$\Rightarrow \int (9+12x+4x^2)^{10}\,dx = \int \left((3+2x)^2\right)^{10} dx = \int (3+2x)^{20}\,dx$$

$$= \frac{(3+2x)^{21}}{21\cdot 2} + C$$

$$\Rightarrow \int \tan^2 x \, dx = \int \left(\frac{1}{\cos^2 x} - 1\right) dx = \tan x - x + C$$

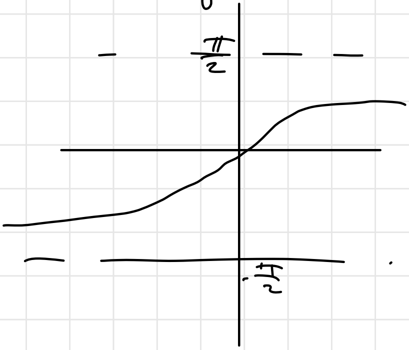
$\cos^2 x + \sin^2 x = 1$
$\sin 2x = 2\sin x \cos x$
$\cos 2x = \cos^2 x - \sin^2 x$
$\cos 2x = 2\cos^2 x - 1$
$\cos 2x = 1 - 2\sin^2 x$
$1 + \tan^2 x = \frac{1}{\cos^2 x}$
$1 + \cot^2 x = \frac{1}{\sin^2 x}$

$$\Rightarrow \int \cos^2 x \, dx = \int \Big(\left(\frac{\cos 2x + 1}{2}\right) dx$$

$$\frac{1}{2}\int (\cos 2x + 1) \, dx = \frac{1}{2}\left(\frac{\sin 2x}{2} + x\right) + C$$

$y = \arcsin x$
$\sin \frac{\pi}{6} = \frac{1}{2}$
$\arcsin \frac{1}{2} = \frac{\pi}{6}$

$y = \arctan x$



$f(x)$	$\int f(x)\,dx$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x$
$\frac{1}{1+x^2}$	$\arctan x$

$y = x^3 + 5x$

$y' = \frac{dy}{dx} = 3x^2 + 5$

$y = t^7$

$y' = \frac{dy}{dt} = 7t^6$

$$x' = \frac{dx}{dy} = \frac{1}{\boxed{\frac{dy}{dx}}} \rightarrow y' = \frac{1}{y'}$$

y	y'
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\arctan\frac{x}{a} + C$

$y = x^2 \Longleftrightarrow x = \sqrt{y}$

$y' = 2x \qquad x' = \frac{1}{2y}$

$x' = \frac{1}{y'}$

$\frac{1}{2\sqrt{y}} = \frac{1}{2x}$

$y = \arcsin x$

$x = \sin y$

$x' = \cos y = \sqrt{1-\sin^2 y} = \sqrt{1-x^2}$

$y' = \frac{1}{x'} = \frac{1}{\sqrt{1-x^2}}$

$y = \arctan x$

$x = \tan y$

$x' = \frac{1}{\cos^2 x} = 1+\tan^2 x = 1+x^2$

$y' = \frac{1}{x'} = \frac{1}{1+x^2}$

$\Rightarrow \int \frac{1}{1+4x^2}\,dx = \int \frac{1}{1+(2x)^2}\,dx = \frac{\arctan(2x)}{2} + C$

$\Rightarrow \int \sin 2x = \frac{-\cos 2x}{2}$

$\Rightarrow \int \frac{1}{9+25x^2}\,dx = \frac{1}{9}\int \frac{1}{1+\frac{25x^2}{9}}\,dx = \frac{1}{9}\int \frac{1}{1+\left(\frac{5x}{3}\right)^2}\,dx = \frac{\frac{1}{9}\arctan\frac{5x}{3}}{\frac{5}{3}}$

$\Rightarrow \int \frac{1}{a^2+x^2}\,dx = \frac{1}{a^2}\int \frac{1}{1+\left(\frac{x}{a}\right)^2}\,dx = \frac{\frac{1}{a^{\cancel{2}}}\arctan\frac{x}{a}}{\frac{1}{\cancel{a}}}$

$= \frac{1}{a}\arctan\frac{x}{a}$

$\Rightarrow$

תרגול 1

אנטגרל :

פעולה הפוכה של נגזרת

① $f(x) = x^2$

$f'(x) = 2x$

$\int 2x\,dx = x^2 + C$

② $f(x) = x$

$f'(x) = 1$

$\int 1\,dx = x + C$

$f(x) = x + C$

$f'(x) = 1$

חוקים

① $n \neq -1 \quad \int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

② $n \neq -1 \quad \int k x^n\,dx = \frac{k x^{n+1}}{n+1} + C$

③ $\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$

④ $\int \overset{x^{-1}}{\overset{\shortparallel}{\frac{1}{x}}}\,dx = \ln x + C$

⑤ $a \neq -1 \quad \int x^a\,dx = \frac{x^{a+1}}{a+1} + C$

⑥ $\int e^x\,dx = e^x + C$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C$$ ⑦

⑧

$f(x) = \sin x$

$f'(x) = \cos x$

$$\int \cos x \, dx = \sin x + C$$

⑨

$f(x) = \cos x$

$f'(x) = -\sin x$

$$\int \sin x \, dx = -\cos x + C$$

תרגילים

חשבו :

$$\int 3x^4 - 5x^3 + 2 \, dx = \frac{3x^5}{5} - \frac{5x^4}{4} + 2x + C$$ ①

②
$$\int -x^5 + 4x^4 - 3x - 6 \, dx = \frac{-x^6}{6} + \frac{4x^5}{5} - \frac{3x^2}{2} - 6x + C$$

$$\int -4x^6 + 3x^4 - x^2 - x \, dx = \frac{-4x^7}{7} + \frac{3x^5}{5} - \frac{x^3}{3} - \frac{x^2}{2} + C$$ ③

תרגול

$$\int \sqrt{x} \, dx = \int x^{\frac{1}{2}} \, dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$\int \frac{1}{3\sqrt{x}} \, dx = \int \frac{1}{3} x^{-\frac{1}{2}} \, dx$$ ①

$$= \frac{\frac{1}{3} x^{\frac{1}{2}}}{\frac{1}{2}}$$

②

$$\int \frac{5x^2 - 3x - 1}{4\sqrt[3]{x}}$$ $x^{\frac{1}{3}}$

$$= \int \frac{1}{4} \cdot (5x^2 - 3x - 1) \cdot x^{-\frac{1}{3}}$$

$$\frac{6}{3} - \frac{1}{3} = \frac{5}{3}$$

$$= \int \frac{1}{4} (5x^{\frac{5}{3}} - 3x^{\frac{2}{3}} - x^{-\frac{1}{3}} \quad dx$$

$$-\frac{1}{3} + \frac{3}{3}$$

✓ $$= \frac{1}{4} \left(\frac{5x^{\frac{8}{3}}}{\frac{8}{3}} - \frac{3x^{\frac{5}{3}}}{\frac{5}{3}} - \frac{x^{\frac{2}{3}}}{\frac{2}{3}} \right) + C$$

③

$$\int \frac{5x^3 - x^{\frac{4}{3}}}{x^2} - \frac{x^2 - 1}{x^3} \quad dx$$

u

$$\int 5x - x^{\frac{1}{3}} - x^{-1} + x^{-3} \quad dx$$

$$= \frac{5x^2}{2} - \frac{x^{\frac{4}{3}}}{\frac{4}{3}} - \ln(x) + \frac{x^{-2}}{-2} + C$$

$$F(x) = 5^x$$

$$f'(x) = 5^x \ln 5$$

$$5^x = \frac{f'(x)}{\ln 5}$$

$$\int 5^x \, dx = \frac{5^x}{\ln 5} + C$$

תרגיל

$$\int \frac{2^x + 4^x}{6^x} \, dx = \int \frac{2^x}{6^x} + \frac{4^x}{6^x} \, dx =$$

$$= \int \frac{1}{3^x} + \left(\frac{2}{3}\right)^x \, dX = \frac{\left(\frac{1}{3}\right)^x}{\ln\left(\frac{1}{3}\right)} + \frac{\left(\frac{2}{3}\right)^x}{\ln \frac{2}{3}} + C$$

תרגיל חמש :

①

$$\int \frac{2x^3-3x}{x^4} - \frac{3}{\cos^2 x} + 4\sin x \, dx$$

$$= \int 2 \cdot x^{-1} - 3x^{-3} - 3 \cdot \frac{1}{\cos^2 x} + 4\sin x \, dx$$

$$= 2\ln x - \frac{3x^{-2}}{-2} - 3\tan x - 4\cos x + C \checkmark$$

②

$$\int \frac{3x^5-3x^2}{x^3} - \frac{1}{2\sin^2 x} \, dx$$

$$= \int 3x^2 - 3x^{-1} - \frac{1}{2} \cdot \frac{1}{\sin^2 x} \, dx$$

$$= \frac{3x^3}{3} - 3\ln x - \frac{1}{2}\cot x + C$$

|

$x = tg\, g\, (x$

$$g(x) = \operatorname{arctg} x$$

$$x = tg\, g(x)$$

$$1 = \frac{1}{\cos^2 g(x)}\, g'(x)$$

$$g'(x) = \cos^2 g(x)$$

$$2 + tg^2 x = \frac{1}{\cos^2 x}$$

$$\cos^2 x = \frac{1}{1 + \tan^2}$$

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הרצאה 2

אנטגרציה בחלקים

$$(u\cdot v)' = u'\cdot v + v'\cdot u$$

$$u'\cdot v = (u\cdot v)' - v'\cdot u$$

$$\int u'\cdot v = \int (u\cdot v)' - \int v'\cdot u$$

$$\boxed{\int u'v = u\cdot v - \int v'\cdot u}$$

→ $\int \underset{v}{x} \cdot \underset{u'}{e^x}\, dx = e^x\cdot x - \int 1\cdot e^x\, dx = e^x\cdot x - e^x + c$

$u = e^x \qquad v = x$

$u' = e^x \qquad v' = 1$

→ $\int \underset{u'}{x}\cdot \underset{v}{e^x}$

$u = \frac{x^2}{2} \qquad v = e^x$

$u' = x \qquad v' = e^x$

מסובך !!

$$\rightarrow \int \underset{u'}{x \sin x}\, dx = -x\cos x - \int 1 \cdot -\cos x \, dx =$$

$$-x\cos x + \sin x + C$$

$u = -\cos x \quad v = x$

$u' = \sin x \quad v' = 1$

$$\rightarrow \int (3x^2 + 5x) \cdot \underset{u'}{e^{7x}}\, dx = \frac{e^{7x}}{7}(3x^2+5x) - \int (6x+5) \cdot \frac{e^{7x}}{7}\, dx$$

$$= \frac{e^{7x}}{7}(3x^2+5x) - \frac{1}{7}\underbrace{\int (6x+5) e^{7x}\, dx}_{\circledast}$$

$$\circledast \quad \frac{e^{7x}}{7}(6x+5) - \int \frac{6 \cdot e^{7x}}{7} dx =$$

$$= \frac{e^{7x}}{7}(6x+5) - \frac{6}{7}\int e^{7x} dx$$

$$\boxed{= \frac{e^{7x}}{7}(6x+5) - \frac{6}{7} \cdot \frac{e^{7x}}{7}}$$

$u = \dfrac{e^{7x}}{7} \quad v = 3x^2+5x$

$u' = e^{7x} \quad v' = 6x+5$

$u = \dfrac{e^{7x}}{7} \quad v = 6x+5$

$u' = e^{7x} \quad v' = 6$

$$\boxed{= \frac{e^{7x}}{7}(3x^2+5x) - \frac{1}{7}\left[\frac{e^{7x}}{7}(6x+5) - \frac{6}{7} \cdot \frac{e^{7x}}{7}\right] + C}$$

סיבה טופית.

תרגיל 6 :

$$\int (5x^2-9x+3)\cdot \underset{u'}{e^{2x}}\,dx = \frac{e^{2x}}{2}\cdot(5x^2-9x+3) - \int (10x-9)\cdot\frac{e^{2x}}{2}\,dx$$

$u = \frac{e^{2x}}{2}$ $\quad v = 5x^2-9x+3$

$u' = e^{2x}$ $\quad v' = 10x-9$

$$= \frac{e^{2x}}{2}\left(5x^2-9x+3\right) - \frac{1}{2}\int (10x-9)\underset{⊛}{\cdot} e^{2x}\,dx =$$

$$\int 10x-9\cdot \underset{u'}{e^{2x}}\,dx = (10x-9)\cdot\frac{e^{2x}}{2} - \int \overset{5}{\cancel{10}}\cdot\frac{e^{2x}}{\cancel{2}}\,dx$$

$u = \frac{e^{2x}}{2}$ $\quad v = 10x-9$

$u' = e^{2x}$ $\quad v' = 10$

$$\boxed{= (10x-9)\cdot\frac{e^{2x}}{2} - \frac{5e^{2x}}{2} + C} \quad ⊛$$

$$\frac{e^{2x}}{2}\left(5x^2-9x+3\right) - \frac{1}{2}\left[(10x-9)\frac{e^{2x}}{2} - \frac{5e^{2x}}{2}\right] + C$$

תרגיל 3 :

$\int x^5 \cdot \ln x \, dx$

$u =$ $v = x^5$

$u' = \ln x$

✗

$\int x^5 \cdot \ln x \, dx = \frac{x^6}{6} \cdot \ln x - \int \frac{1}{x} \cdot \frac{x^6}{6} = \hookleftarrow$

$u = \frac{x^6}{6}$ $v = \ln x$

$u' = x^5$ $v' = \frac{1}{x}$

$= \frac{x^6}{6} \ln x - \frac{1}{6} \int x^5 \, dx =$

$$\boxed{= \frac{x^6}{6} \ln x - \frac{1}{6} \cdot \frac{x^6}{6} + C}$$

ג.ס

תרגיל 4 :

$\int x^{-7} \ln x \, dx = x^{-7} \ln x - \int \frac{1}{x} \cdot \frac{x^{-6}}{-6} \, dx = x^{-7} \cdot \ln x + \frac{1}{6} \int x^{-7} \, dx$

$u = \frac{x^{-6}}{-6}$ $v = \ln x$

$u' = x^{-7}$ $v' = \frac{1}{x}$

$$= \boxed{x^{-7} \cdot \ln x + \frac{1}{6} \cdot \frac{x^{-6}}{-6} + C}$$

תרגיל 5 :

$$\int 1 \ln x\,dx = x\cdot\ln x - \int \frac{1}{\cancel{x}}\cdot \overset{1}{x}\,dx = x\ln x - \int 1\,dx =$$

$u = x \qquad v = \ln x$

$u' = 1 \qquad v' = \frac{1}{x}$

$$= x\ln x - x + C$$

תרגיל 6 :

$$\int e^x\cdot\cos x\,dx = e^x\cdot\cos x - \int \underbrace{-\sin x\cdot e^x}_{u'}\,dx$$

$\circledast$

$u = e^x \qquad V = \cos x$

$u' = e^x \qquad V' = -\sin x$

$$= e^x\cos x + e^x\sin x - \int e^x\cos x\,dx$$

$$\int \sin x\, e^x = e^x\sin x - \int e^x\cos x\,dx$$

$u = e^x \qquad v = \sin x$

$u' = e^x \qquad v' = \cos x$

$$2\int e^x\cos x\,dx = e^x\cos x + e^x\sin x$$

$$\int e^x\cos x\,dx = \frac{e^x\cos x + e^x\sin x}{2} + c$$

תרגיל 7 :

$$\int e^{-3x}\sin 7x \, dx = \frac{e^{-3x}}{-3}\sin 7x - \underbrace{\int 7\cos 7x \cdot \frac{e^{-3x}}{-3} dx}_{\circledast}$$

$$u = \frac{e^{-3x}}{-3} \qquad v = \sin 7x$$

$$u' = e^{-3x} \qquad v' = 7\cos 7x$$

$$\circledast \quad \frac{e^{-3x}}{-3}\sin 7x + \frac{7}{3}\underbrace{\int e^{-3x}\cos 7x \, dx}$$

$$= \frac{e^{-3x}}{3}\sin 7x + \frac{7}{3}\left[\frac{e^{-3x}}{-3}\cos 7x - \frac{7}{3}\int e^{-3x}\sin x 7x \, dx\right]$$

$$\int e^{-3x}\cos 7x = \frac{e^{-3x}}{-3}\cos 7x \int -7\sin 7x \cdot \frac{e^{-3x}}{-3} . \cos 7x - \frac{7}{3}\int e^{-3x}\sin 7x$$

$$\left(1+\frac{49}{9}\right)\int e^{-3x}\sin 7x = \frac{\frac{e^{-3x}}{-3}\sin 7x + \frac{7}{3}\frac{e^{-3x}}{-3}\cos 7x}{1+\frac{49}{9}} + C$$

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הרצאה 3

$$\int (x+2)\ln(x+2)\,dx$$

$v = \ln(x+2)$

$u' = x+2$ $\qquad$ $v' = \frac{1}{x+2}$

$$\int \sin(\ln x)\,dx = x\sin\ln x - \int \cos(\ln x)\cdot\frac{1}{\cancel{x}}\cdot\cancel{x}\,dx$$

$u = x$ $\qquad$ $v = \sin(\ln x)$

$u' = 1$ $\qquad$ v' $\cos(\ln x)\cdot\frac{1}{x}$

$$x\sin(\ln x) - \left(x\cos(\ln x) + \int\sin(\ln x)\right).$$

$$\int\sin(\ln x)\,dx = \boxed{\frac{x\sin x\left(\ln x - x\cos\ln x\right)}{2} + C}$$

$$\int\cos(\ln x)\,dx = x\cos\ln x + \int +\sin(\ln x)\cdot\frac{1}{\cancel{x}}\cdot\cancel{x}$$

$u = x$ $\qquad$ $v = \cos(\ln x)$

$u' = 1$ $\qquad$ $v' = -\sin(\ln x)\,\frac{1}{x}$

$$a^2-b^2=(a+b)(a-b)$$

$$(a+b)^2=a^2+2ab+b^2$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$a^3+b^3=(a+b)(a^2+ab+b^2)$$

$$ax^7+bx+c=a(x-x_1)(x-x_2)$$

x_1,x_2 פתרונות במספר תרגילים

$$\int\frac{1}{x-5}\,dx=\ln(x-5)\,dx$$

$$\int\frac{1}{3x-5}\,dx=\frac{\ln(3x-5)}{3}+C$$

$$\int\frac{1}{(x-3)\cdot(x-7)}$$

$$\frac{1}{(x-3)\cdot(x-7)}=\frac{A^{(x-7)}}{(x-3)}+\frac{B^{(x-3)}}{(x-7)}$$

$$1=A(x-7)+B(x-3)$$

$$ax^2-3x-5=7x^2+bx+c$$

$a=7$

$b=-3$

$c=-5$

$1 = A(x-7) + B(x-3)$

$1 = x(A+B) + (-3B-7A)$

$A + B = 0$

$-7A - 3B = 1$

$A - \frac{7A}{3} \frac{1}{3} = 0$

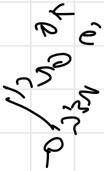
$-\frac{7A-1}{3} = \frac{3B}{3}$

$3A - 7A = 1$

$-4A = 1$

$= B$

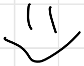
$\boxed{A = -\frac{1}{4}}$



$x = 3 \quad 1 = A(3-7) \Rightarrow A = -\frac{1}{4}$

$x = 7 \quad 1 = 4b \quad B = \frac{1}{4}$

$\frac{1}{(x-3)(y-7)} = \frac{1}{4} \cdot \frac{1}{x-3} + \frac{1}{4} \frac{1}{x-7}$



$$\int \frac{1}{(x-3)(x-7)} dx = \frac{-1}{4} \int \frac{1}{x-3} dx + \frac{1}{4} \int \frac{1}{x-7} dx$$

$$= \frac{5}{4} \ln(x-3) + \frac{1}{4} \ln(x-7) + C$$

$$\frac{7}{x(x+5)} = \frac{\overset{(x+5)}{A}}{x} + \frac{\overset{x}{B}}{x+5}$$

$7 = A(x+5) + B(x)$

$x = 0 \quad 7 = 5A \quad A = \frac{7}{5}$

$x - 5 \quad 7 = -5B \quad B = \frac{-7}{5}$

$$\int \frac{7}{x(x+3)} dx = \frac{7}{5} \int \frac{1}{x} dx - \frac{7}{5} \int \frac{1}{x+5} dx$$

$$\boxed{= \frac{7}{5} \ln x - \frac{7}{5} \ln(x+5) + C}$$

$$\int \frac{3x-4}{(x+4)(x-2)} dx$$

$$\frac{3x-4}{(x+4)(x-2)} = \frac{A}{x+4} + \frac{B}{(x-2)}$$

$-2A + 4B$

$$3x-4 = A(x-2) + B(x+4)$$

$$= \frac{8}{3}\int \frac{1}{x+4} dx + \frac{1}{3}\int \frac{1}{x-2} dx$$

$$3x-4 = x(A+B) + 4B-2A$$

$3 = A+B$

$-4 = 4B-2A$

$2: \quad \cancel{-}2A = 4B+4$

$\boxed{A = 2B+2}$

$3 = 2B+2+B$

$3 = 3B+2$

$\boxed{\frac{1}{3} = B}$

$\boxed{A = \frac{8}{3}}$

$$= \frac{8}{3} \ln(x+4) + \frac{1}{3} \ln(x-2) + C$$

אפשר לבדוק

ק^2 !!

המטרה שהחזקה במכנה גדולה מהחזקה במונה

ובמכנה מחולק לשורשים ממשיים שונים אז

אפשר לפרק את הביטוי לצורת.

$$\int \frac{2x^2 - 5x + 9}{x^3 - 9x}\,dx \qquad \frac{2x^2 - 5x + 9}{x(x-3)(x+3)} = \frac{\overset{(x-3)(x+3)}{A}}{x} + \frac{\overset{x(x+3)}{B}}{x-3} + \frac{\overset{x(x-3)}{C}}{x+3}$$

$$x(x-3)(x+3) \qquad 2x^2 - 5x + 9 = A(x-3)(x+3) + Bx(x+3) + Cx(x-3)$$

$x = 0$ $\qquad 9 = -9A \qquad A = -1$

$x = 3$ $\qquad 12 = 18B \qquad B = \frac{2}{3}$

$x = -3$ $\qquad 42 = 18C \qquad C = \frac{7}{3}$

$$= -1 \int \frac{1}{x}\,dx + \frac{2}{3}\int \frac{1}{x-3}\,dx + \frac{7}{3}\int \frac{1}{x+3}\,dx$$

$$= \boxed{-\ln(x) + \frac{2}{2}\ln(x-1) + \frac{7}{3}\ln(x+3) + C}$$

$$\int \frac{8x-3x^2+2}{(x^2-4)(x+5)}$$

$$\frac{-3x^2+8x+2}{(x^2-4)\,x+5} = \frac{A}{(x-2)} + \frac{B}{(x+2)} + \frac{C}{x+5}$$

$-12-16+2$

$$-3x^2+8x+2 = (x+5)(x+2)A + (x-2)(x+5)\cdot B + (x-2)(x+2)C$$

-3

$x=-5$ $\quad -113 = 21C \quad C = \frac{-113}{21}$

$x=2$ $\quad 6 = 28A \quad A = \frac{3}{14}$

$x=-2$ $\quad -26 = -12B \quad B = \frac{13}{6}$

$$= \frac{3}{14}\int \frac{1}{x-2}\,dx + \frac{13}{6}\int \frac{1}{x+2}\,dx - \frac{113}{21}\int \frac{1}{x+5}\,dx$$

$$\frac{3}{14}\ln(x-2) + \frac{13}{6}\ln(x+2) - \frac{113}{21}\ln(x+5) + C$$

$$\frac{x^3-9}{x-2}$$

x^2+2x+4

$x^3-9 \quad | \quad x-2$

x^3-2x^2

$2x^2-9$

$2x^2-4x$

$4x-9$

$-\ 4x-8$

-1

$$\int \frac{3x+1}{(x-2)^3}\, dx$$

$1/$

$(x-2)^2 \qquad (x-2)\ 1$

$$\frac{3x+1}{(x-2)^3} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3}$$

$3x+1 = A(x-2)^2 + B(x-2) + C$

$x=2 \qquad 7=C \qquad C=7$

$$= 3\int \frac{1}{(x-2)^2}dx \quad dx \quad + 7\int \frac{1}{(x-2)^3}dx$$

$x=0 \qquad 1=4A-2B+C$

$x=3 \qquad 10=A+B+C$

$$= \frac{3(x-2)^{-1}}{-1} + \frac{7(x-2)^{-2}}{-2}$$

$x=1 \qquad 4=-A-B+C$

$A=0 \quad B=3$

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תרגול 3

$$\int \frac{2x^2+5x-1}{(x-2)(x+3)(x-4)}dx$$

$$\frac{2x^2+5x-1}{(x-2)(y+3)(x-4)} = \frac{A}{(x-2)} + \frac{B}{(x+3)} + \frac{C}{(x-4)}$$

$8+10-1$ $$2x^2+5x-1 = (x+3)(x-4)A + B(x-2)(x-4) + C(x-2)(x+3)$$

-5 $2\cdot 7$

$x=2$ $17 = -10A$ $A = -1.7$

$x=-3$ $2 = -35B$ $B = -\frac{2}{35}$

$x=4$ $51 = 14C$ $C = \frac{51}{14}$

$$= -1.7\int \frac{1}{x-2}dx - \frac{2}{35}\int \frac{1}{x+3}dx + \frac{51}{14}\int \frac{1}{x-4}dx$$

$$= -1.7\ln(x-2) - \frac{2}{35}\ln(x+3) + \frac{51}{14}\ln(x-4) + C$$

$$\int \frac{3x^2-4x}{(x+2)^3}$$

$$\frac{3x^2-4x}{(x+2)^3} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3}$$

$$3x^2-4x = (x+2)^2A + B(x+2) + C$$

$x=-2$

$x=0$

$x=3$

$$\boxed{20 = C} \qquad \boxed{B = -16}$$

$$0 = \underset{12}{4A} + 2B + \underset{20}{C}$$

$$39 = A - B + C$$

$$\frac{-4A}{2} - \frac{20}{2} = \frac{2B}{2}$$

$$= 3\int \frac{1}{x+2}dx - 16\int \frac{1}{(x+2)^2}dx + 20\int \frac{1}{(x+2)^3}dx$$

$$= 3\ln(x+2) - \frac{16(x+2)^{-1}}{-1}$$

$$+ \frac{20(x+2)^{-2}}{-2}$$

$$\boxed{-2A - 10 = B}$$

$$39 = A + 2A + \overset{30}{10 + 20}$$

$$\frac{9}{3} = \frac{3A}{3}$$

$$\boxed{3 = A}$$

$$3x^2 - 7x + 1$$

$$3x^5 - 7x^4 + 4x^3 - 9x + 2 \quad \big| \quad x^3 + x + 2$$

$$- \quad 3x^5 + 3x^3 + 6x^2$$

$$-7x^4 + x^3 - 6x^2 - 9x + 2$$

$$- \quad -7x^4 - 7x^2 - 14x$$

$$x^3 + x^2 + 5x + 2$$

$$x^3 + x + 2$$

$$x^2 + 4x$$

$$\frac{3x^5 - 7x^4 + 4x^3 - 3x + 2}{x^3 + x + 2} = 3x^2 - 7x + 1 + \frac{x^2 + 4x}{x^3 + x + 2}$$

$$x^2-x+2$$

$$x^2+x+2 \;\Big|\; x+4$$

הוא
מחלק אותנו.

$\vdots$

$$\int \frac{-3x^4-x^3+4}{x^3-25x}\,dx$$

הכל

$$-3x-1$$

$$-3x^4-x^3+4 \;\Big|\; x^3-25$$

$$-\;\; -3x^4+75x$$

$$-x^3-75x+4$$

$$-x^3+25$$

$$-75x-21$$

$$-3x-1+\frac{-75x-21}{x^3-25}$$

175

$$=\int(-3x-7)\,dx+\int\frac{75x^2-175x+4}{x(x-5)(x+5)}\,dx$$

ככי
פלאחנו.

$$=\int(-3x-7)\,dx \;-$$

3/4/22

הרצאה 4

כלל מהנוסחאות

$$\int \frac{1}{x^2+1}\,dx = \arctan x + C \qquad \frac{1}{1+x^2}$$

כלל נוסף מהנוסחאות

$$\int \frac{1}{x^2+a^2}\,dx = \ ? \ \underline{\underline{\text{נוכיח}}} = \frac{1}{a}\arctan\frac{x}{a} + C$$

$$= \frac{1}{a^2}\int \frac{1}{\left(\frac{x}{a}\right)^2+1}\,dx = \frac{\frac{1}{a^{\cancel{2}}}\arctan\frac{x}{a}}{\cancel{\frac{1}{a}}} + c = \frac{1}{a}\arctan\frac{x}{a} + c$$

$$\Rightarrow \quad y = \ln(f(x)) \qquad y' = \frac{f(x)}{f(x)} \qquad \Longleftrightarrow \qquad \int \frac{f'(x)}{f(x)}\,dx = \ln(f(x)) + C$$

$$\Rightarrow \quad \int \frac{2x}{x^2+1}\,dx = \ln(x^2+1) + C$$

$$\Rightarrow \quad \int \frac{3x}{x^2+1}\,dx = \frac{3}{2}\int \frac{2x}{x^2+1}\,dx = \frac{3}{2}\ln(x^2+1)+c$$

$$\Rightarrow \int \frac{x+1}{x^2+9} dx = \int \frac{x}{x^2+9} + \frac{1}{x^2+9} dx$$

$$\frac{1}{2}\int \frac{2x}{x^2+9} dx + \int \frac{1}{x^2+9} dx$$

$$\boxed{\frac{1}{2}\ln(x^2+9) + \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C}$$

$$\Rightarrow \int \frac{x^2}{x^3+5} dx = \frac{1}{3}\int \frac{3x^2}{x^3+5} dx = \frac{1}{3}\ln(x^3+5) + C$$

$$\Rightarrow \int \frac{1}{x\ln x} dx = \int \frac{\frac{1}{x}}{\ln(x)} dx = \ln(\ln(x)) + C$$

$$\Rightarrow \int \frac{2x+3}{(x-2)(x^2+1)} dx$$

$$\frac{2x+3}{(x-2)(x^2+1)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+1}$$

מכיוון שהוא
לכן מתפצל

$$= \frac{A}{x-2} + \frac{Bx}{x^2+1} + \frac{C}{x^2+1} \dots$$

$$2x+3 = A(x^2+1) + Bx(x-2) + C(x-2)$$

$$= A\int \frac{1}{x-2} dx + B\int \frac{x}{x^2+1} dx + C\int \frac{1}{x^2+1} dx$$

$$= A\ln(x-2) + \frac{B}{2}\ln(x^2+1) + C\arctan x + C$$

$x=2$ $\quad 7 = 5A$

$x=0$ $\quad 3 = A - 2C$

$x=3$ $\quad 9 = 10A + 3B + C$

⋮

תרגיל 1

$$\int \frac{x^2-3x+5}{x^3+x}\,dx = \int \frac{x^2-3x+5}{x(x^2+1)}\,dx$$

$$= 5\int \frac{1}{x}\,dx - \frac{10}{2}\int \frac{2x}{x^2+1}\,dx - 3\int \frac{1}{x^2+1}\,dx$$

$$\frac{x^2-3x+5}{x(x^2+1)} = \frac{A}{x} + \frac{Bx}{x^2+1} + \frac{C}{x^2+1}$$

$$x^2-3x+5 = (x^2+1)A + Bx^2 + Cx$$

$$= 5\ln(x) - 5\ln(x^2+1) - 3\arctan(x) + C$$

$x=0$ $\quad 5 = A$ $\quad \boxed{A=5}$

$x=1$ $\quad 3 = \overset{10}{\cancel{2A}} + B + C \Rightarrow -7 - \overset{-3}{\cancel{C}} = B$

$x=2$ $\quad 3 = \overset{25}{\cancel{5A}} + 4B + 2C =$ $\quad \boxed{-10 = B}$

$3 = 25 \overset{-3}{-} 28 - 4C + 2C$

$6 = -2C$ $\quad \boxed{C = -3}$

תרגיל 2

$$\int \frac{1}{x^2-4x+5}\,dx =$$

$$x^2-4x+5 = x^2-4x+4+1 = (x+2)^2+1$$

$$= \int \frac{1}{(x-2)^2+1}\,dx = \arctan(x-2) + C$$

תרגיל 3

$$\int \frac{x^3-7x^2+15}{x^2-6x+13}\,dx = \int (x-1)+\frac{-19x+28}{x^2-6x+13}\,dx$$

$$\begin{array}{r|l} x-1 & \\ \hline x^3-7x^2+15 & x^2-6x+13 \\ -\;x^3-6x^2+13x & \\ \hline -x^2-13x+15 & \\ -\;-x^2+6x-13 & \\ \hline -19x+28 & \end{array}$$

$$\int (x-1)\,dx + \int \frac{-19x+28}{x^2-6x+13}\,dx$$

60 $\frac{30}{2}$ $3*19$

$-19x+28$ $-\frac{19}{2}(2x-2\cdot 3)$

$\frac{f'(x)}{f(x)}$

$$= \frac{(x-1)^2}{2} +$$

$$\int \frac{-19x+28}{x^2-6x+13}\,dx = \int \frac{-19}{2}\left(\frac{2x-6}{x^2-6x+13}\right) - \frac{29}{x^2-6x+13}$$

$$= \frac{-19}{2}\int \frac{2x-6}{x^2-6x+13}\,dx - 29\int \frac{1}{(x-3)^2+2^2}\,dx =$$

[illegible]

$$= \frac{-19}{2}\ln\left(x^2-6x+13\right) - 29\cdot\frac{1}{2}\arctan\left(\frac{x-3}{2}\right) + \frac{x^2}{2} - x + C$$

תרגיל 24

$$\int \frac{1}{x^4-16}\,dx$$

$$\frac{1}{x^4-16} = \frac{1}{(x^2)^2-4^2} = \frac{1}{(x^2-4)(x^2+4)} = \frac{1}{(x-2)(x+2)\ x^2+4}$$

$$= \frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx}{x^2+4} + \frac{D}{x^2+4} \qquad 1 = \text{- - - - -}$$

$x=2$ $\quad 1 = 32A$ $\quad \boxed{A = \frac{1}{32}}$

$x=-2$ $\quad 1 = -32B$ $\quad \boxed{B = \frac{-1}{32}}$

$x=0$ $\quad 1 = \frac{8}{32} + \frac{8}{32} - 4D$ $\quad 4D = -\frac{1}{2}$ $\quad \boxed{D = -\frac{1}{8}}$

$x=1$ $\quad 1 = 15A - 5B - 3C - 3D$ $\quad \boxed{C = 0}$

$$= \frac{1}{32}\int \frac{1}{x-2}\,dx - \frac{1}{32}\int \frac{1}{x+2}\,dx - \frac{1}{8}\int \frac{1}{x^2+4}\,dx$$

$$= \boxed{\frac{1}{32}\ln(x-2) - \frac{1}{32}\ln(x+2) - \frac{1}{8}\cdot\frac{1}{2}\cdot\arctan\left(\frac{x}{2}\right) + C}$$

3/4/22

תרגול 4

$$\int \frac{2x^2-1}{(4x-1)(x^2+1)}\,dx = \frac{-14}{17}\int\frac{1}{4x-1}\,dx + \frac{6}{17}\int\frac{2x}{x^2+1}\,dx + \frac{3}{17}\int\frac{1}{x^2+1}\,dx =$$

$$\frac{2x^2-1}{(4x-1)(x^2+1)} = \frac{A}{4x-1} + \frac{Bx}{x^2+1} + \frac{C}{x^2+1}$$

$$= \frac{-14}{17}\frac{\ln(4x-1)}{4} + \frac{6}{17}\ln(x^2+1) + \frac{3}{17}\arctan(x) + C$$

$\vdots$

$$\int \frac{x^4+6x^3+10x^2+x}{x^2+6x+10}\,dx$$

$$\begin{array}{r|l} x^2 & \\ \hline x^4+6x^3+10x^2+x & x^2+6x+10 \\ -\ x^4+6x^3+10x^2 & \\ \hline x & \end{array}$$

$$= \int x^2 + \frac{x}{x^2+6x+10}\,dx \Rightarrow \int x^2\,dx + \int \frac{x}{x^2+6x+10}\,dx$$

①

$$= \frac{x^3}{3} +$$

①

$$\int \frac{x}{x^2+6x+1\underset{\wedge}{0}}\,dx =$$

$$(x+2)(x+4)+2$$

10/4/22

הרצאה 5

$$\Rightarrow \int 2x\,(x^2+5)^{13}\,dx = \int 2x \cdot t^{13} \cdot \frac{dt}{2x} = \int t^{13}\,dt = \frac{t^{14}}{14}$$

$$= \frac{(x^2+5)^{24}}{14} + C$$

$t = x^2+5$ $\qquad dx = \frac{dt}{2x}$

$dt = 2xdx$

$$\int x^2(2x^3+7)^{10}\,dx = \int x^2 \cdot x^{10} \cdot \frac{dt}{6x^2} = \frac{1}{6}\int t^{13}\,dt$$

$$= \frac{1}{6}\,\frac{t^{11}}{11} + C = \boxed{\frac{1}{6} \cdot \frac{(2x^3+7)^{11}}{11} + C}$$

$$\Rightarrow \int x^5(2x^3+1)^7\,dx = \int x^5 \cdot t^7 \cdot \frac{dt}{6x^2} = \frac{1}{6}\int x^3 \cdot t^7\,dt$$

$t = 2x^3+1$ $\qquad 6x^2dx$

$dt =$ $\qquad dx/6x^3$

→ $\int \frac{1}{x\ln^2 x}\,dx = \int \frac{1}{x\,t}\cdot x\,dt = \int \frac{1}{t}\,dx \int = \ln(t)$

$t = \ln(x)$

$1\,dt = \frac{1}{x}\,dx$

$dx = x\,dt$

$= \boxed{\ln(\ln x) + C}$

→ $\int \frac{1}{x\ln\cdot\ln(\ln x)}\,dx$

$t = \ln(\ln x)\,dx$

$dt = \frac{1}{\ln(x)}\cdot\frac{1}{x}\,dx$

$= \boxed{\ln(\ln(\ln x)) + C}$

$\boxed{\sin^2 x + \cos^2 x = 1}$

$\int \sin^7 x \cos^5 x\,dx$

$t = \sin x$

$1\,dt = \cos x\,dx$

$= \int t^7 \cos^{\overset{4}{\cancel{5}}} x \cdot \frac{dt}{\cancel{\cos x}}$

$dx = \frac{dt}{\cos x}$

$= \int t^7 (1-t^2)^2\,dx = \int t^7 (1-2t^2+t^4)\,dt$

$\cos^4 x = (1-t^2)^2$

$= \int t^7 - 2t^9 + t^{11}\,dt = \boxed{\frac{t^8}{8} - \frac{2t^{10}}{10} + \frac{t^{12}}{12} + c =}$

$t = \sin x$? :3)

✓

$\Rightarrow \int \sin^5 x \cdot \cos^4 x \, dx$

$\sin^6 x = (1-t^2)^2$

$t = \cos x$

$dt = -\sin dx$

$dx = \dfrac{dt}{-\sin x}$

$$= \int \sin^{\overset{4}{\cancel{5}}} x \cdot t^4 \frac{dt}{\cancel{-\sin x}} = -\int (1-t^2)^2 t^4 dx$$

$$= -\int (1 2t^2 + t^4)\, t^4 \cancel{dx} = \int t^4 - 2t^6 + t^8 \, dt$$

$$= \frac{-t^5}{5} + \frac{2t^7}{7} \; \frac{-t^9}{9} =$$

$$\Rightarrow \int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} dx = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + C$$

$\Rightarrow \int \sin^4 x \cos^5 x$

[illegible] →

$\sin \cos x = \dfrac{\sin 2x}{2}$

$$= \int (\sin x \cos x)^2 \sin^2 x \, dx =$$

$$= \int \frac{1}{4} \sin^2 2x \; \frac{(1-\cos 2x)}{2} dx$$

$$= \frac{1}{8} \int \sin^2 2x \, (1-\cos(2x)) \, dx = \frac{1}{8} \int \sin^2 \overset{\nearrow}{2} \, dx$$

$$-\frac{1}{8}\int \sin^2 2x \cos 2x \, dx = \frac{1}{4}\left[x - \frac{\sin 4x}{4}\right]$$

$$\boxed{= \frac{1}{8}\frac{\sin^3 2x}{2} + C}$$

$$\Rightarrow \int \sqrt{1-x^2}\, dx = \int (\underbrace{1-\sin^2 t)}_{\cos t}) \cos t \, dt = \quad x = \sin t$$

$$1 \, dx = \cos t \, dt$$

$$= \int \cos^2 x \, t \, dt = \int \frac{1+\cos 2t}{2} \, dt$$

$$= \frac{1}{2}\left[t + \frac{\sin 2t}{2}\right] = \frac{1}{2}\left[\arcsin x + \frac{\sin(2\arcsin x)}{2}\right] + C$$

arcSin x ?

[illegible]

$$\Rightarrow \frac{x-1}{\sqrt{1-x^2}} \, dx = \int \frac{x}{\sqrt{1-x^2}} \, dx - \int \frac{1}{\sqrt{1-x^2}} \, dx =$$

$$t = \sqrt{1-x^2}$$

$$dt = \frac{1}{\cancel{2}\sqrt{1-x^2}} \cdot \cancel{-2}x \, dx$$

$$dx = \frac{-dt\sqrt{1-x^2}}{x}$$

$$= \int \frac{x}{\sqrt{1-x^2}} \, dx = \int \frac{x}{\sqrt{1-x^2}} \cdot \frac{dt\sqrt{1-x^2}}{-x} dt$$

$$= \int -1 \, dt = -t$$

$$\boxed{= -\sqrt{1-x^2} - \arcsin x + C}$$

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הרצאה 6

תזכורת :

$$\int \sin^5 3x \cos^8 3x \, dx = \int \sin^{\cancel{8}\,4} 3x \cdot t^8 \cdot \frac{dt}{\cancel{-3\sin 3x}} =$$

$t = \cos 3x$

$1\,dt = -3\sin 3x \; dx$

$$\boxed{dx = \frac{dt}{-8\sin 3x}}$$

$$\sin^4 3x = \Big(\underbrace{\sin^2 x\, 3x}_{}\Big)^2 = \Big(1 - \cos^2 3x\Big)^2$$

$$= -\frac{1}{3}\int (1 - t^2)\, t^8 \, dt = -\frac{1}{3}\int (1 - 2t^2 + t^4)\, t^8 \, dt$$

$$= -\frac{1}{3}\int t^8 - 2t^{10} + t^{12} \, dt = -\frac{1}{3}\left[\frac{t^9}{9} - \frac{2t^{11}}{11} + \frac{t^{13}}{13}\right]$$

$$= -\frac{1}{3}\Big\{ \cos^3 \qquad\qquad + C$$

להציב במקום t

$$\Rightarrow \int \sin^4 x \, dx =$$

$$\sin^4 x = \left(\sin^2 x\right)^2 = \left(\frac{1-\cos 2x}{2}\right)^2$$

$$= \frac{1}{4}\left[1-2\cos 2x + \cos^2 2x\right]$$

$$= \frac{1}{4}\left[1-2\cos 2x + \frac{1+\cos 4x}{2}\right]$$

$$= \frac{1}{4}\int\left(1-2\cos 2x + \frac{1}{2} + \frac{\cos 4x}{2}\right)dx$$

$$\boxed{= \frac{1}{4}\left[x - \frac{\sin 2x}{2} + \frac{1}{2}x + \frac{\sin 4x}{4\cdot 2}\right] + C}$$

$$\Rightarrow \int R\left(\sin x, \cos x\right) \qquad t = \tan\frac{x}{2}$$

$$\boxed{\begin{gathered} 1+\tan^2\frac{x}{2} = \frac{1}{\cos^2\frac{x}{2}} \\ \cos^2\frac{x}{2} = \frac{1}{1+\tan^2\frac{x}{2}} = \frac{1}{1+t^2} \end{gathered}}$$

$$\frac{2t}{1+t^2} = \Rightarrow \sin x = \frac{2\sin\frac{x}{2}\cdot\cos^2\frac{x}{2}}{\cos\frac{x}{2}} = 2t\cdot\cos^2\frac{x}{2}$$

$$\frac{1-t^2}{1+t^2} = \Rightarrow \cos x = 2\cos^2\frac{x}{2} - 1 = 2\left(\frac{1}{1+t^2}\right) - 1 = \frac{2}{1+t^2} - 1 = \frac{2-1-t^2}{1+t^2} = \frac{1-t^2}{1+t^2}$$

$$\Rightarrow t = \tan\frac{x}{2}$$

$$\Rightarrow dx = \frac{2dt}{1+t^2} \qquad 1\,dt = \frac{1}{2}\cdot\frac{1}{\cos^2\frac{x}{2}}\,dx = \frac{1}{2}\left[1+\tan^2\frac{x}{2}\right]dx = \frac{1+t^2}{2}\,dx$$

$$\Rightarrow dx = \frac{2dt}{1+t^2}$$

תרגילים:

$$\int \frac{dx}{2\sin x+\cos x+2} = \int \frac{\frac{2dt}{1+t^2}}{2\frac{2t}{2+t^2}+\frac{1-t^2}{1+t^2}+2} = \int \frac{\frac{2}{1+t^2}}{\frac{4t+1-t^2+2+2t^2}{1+t^2}}\, dt$$

$$= \int \frac{2}{t^2+4t+3}\, dt = \int \frac{-1}{t+3}+\frac{1}{t+1}\, dt = -\ln(t+3)+\ln(t+1)$$

$$\frac{2}{t^2+4t+3} = \frac{\overset{(t+1)}{A}}{(t+3)}+\frac{\overset{(t+3)}{B}}{t+1}$$

פתרון סופי

$$\boxed{= -\ln\left(\tan\frac{x}{2}+3\right)+\ln\left(\tan\frac{x}{2}+1\right)+C}$$

$$2 = A(t+1)+B(t+3)$$

$t=-1 \quad 2=2B$

$t=-3 \quad 2=-2A$

$$\boxed{\begin{matrix}B=1\\A=-1\end{matrix}}$$

$$\int \frac{1}{\sin x}\, dx = \int \frac{\frac{\cancel{2}}{\cancel{1+t^2}}dt}{\cancel{\frac{2t}{1+t^2}}} = \int \frac{1}{t}\, dt$$

$$= \ln t = \ln\left(\tan\frac{x}{2}\right)+C$$

$$\int \frac{1}{\cos x}\, dx = \int \frac{1}{\frac{1-t^2}{\cancel{1+t^2}}}\, \frac{2dt}{\cancel{1+t^2}} = \int \frac{2}{1-t^2}\, dt =$$

$$= \frac{2}{1-t^2} = \frac{A}{1-t}+\frac{B}{1+t}$$

$$= \int \frac{1}{1-t}+\frac{-1}{1+t}$$

$$= \frac{\ln(1-t)}{-1} - \ln 1+t$$

יש לו טעות בפתרון...

$$2 = (1+t)A + (1-t)B$$

$t = 1 \qquad 2 = 2A$

$t = -1 \qquad 2 = -2B$

$$\boxed{\begin{array}{c}A=1\\ B=-1\end{array}}$$

⇒ $$\int \frac{7}{8\cos x - 10 - 10\sin x}\,dx = \int \frac{7}{8\,\frac{1-t^2}{1+t^2} + 10 - 10\,\frac{2t}{1+t^2}}\;\frac{2dt}{1+t^2}$$

$$= \int \frac{\dfrac{14\,dt}{\cancel{1+t^2}}}{\dfrac{8-8t^2+10+10t^2-20t}{\cancel{1+t^2}}} = \int \frac{14\,dt}{2t^2-20t+18}$$

$$= \int \frac{7}{t^2-10t+9}\,dt = \frac{7}{8}\int \frac{1}{t-9}\,dt - \frac{7}{8}\int \frac{1}{t-1}\,dt$$

$$= \frac{7}{8}\ln(t-9) - \frac{7}{8}\ln(t-1)$$

$$= \boxed{\frac{7}{8}\ln\left(\tan\frac{x}{2} - 9\right) - \frac{7}{8}\ln\left(\tan\frac{x}{2} - 1\right) + C}$$

האינטגרל המסוים

$$\int f(x)\,dx = F(x)+C$$ אם

$$\int_a^b f(x)\,dx = F(b)-F(a)$$ אז

משמשים [illegible]
[illegible] למצב [illegible]
[illegible] פשוט [illegible] !!

⇒ $$\int_1^3 x^2\,dx = \frac{x^3}{3}\Big|_1^3 = \frac{27}{3}-\frac{1}{3} = \frac{26}{3}$$

⇒ $$\int_1^3 x^2-4x+3\,dx = \frac{x^3}{3}-\frac{\overset{2}{\cancel{4}}x^2}{\cancel{2}}+3x\Big|_1^3$$

$$\underbrace{\frac{3^3}{3}-2\cdot3^2+3\,3}_{0}-\underbrace{\left[\frac{1}{3}-2+3\right]}_{\frac{4}{3}} = 0-\frac{4}{3} = -\frac{4}{3}$$ ✓

⇒ $$\int_{-2}^{2} x^3\,dx = \frac{x^4}{4}\Big| = 0$$

תזכורת

$$\int u'v = u\cdot v-\int v'u$$

⇒ $$\underset{u'}{e^{-3x}}(5x^2\underset{v}{+}7x-2)\,dx =$$

$$u=\frac{e^{-3x}}{-3} \qquad v=5x^2+7x-2$$

$$u'=e^{-3x} \qquad v'=10x+7$$

$$=\frac{e^{-3x}}{-3}(5x^2+7x-2)+\frac{1}{3}\int(10x+7)\cdot e^{-3x}\,dx$$

⇑

$$= \int (10x+7)\, e^{-3x} \qquad = \frac{e^{-3x}}{-3}(10x+7) + \frac{1}{3}\int 10 \cdot e^{-3x}\, dx$$

$$u = \frac{e^{-3x}}{-3} \quad v = 10x+7 \qquad = \frac{e^{-3x}}{-3}(10x+7) + \frac{10}{3}\int e^{-3x}\, dx$$

$$u' = e^{-3x} \quad v' = 10 \qquad = \frac{e^{-3x}}{-3}(10x+7) + \frac{10}{3} \cdot \frac{e^{-3x}}{-3}$$

$$\boxed{\therefore \frac{e^{-3x}}{-3}\left(5x^2+7x-2\right) + \frac{1}{3}\left[\frac{e^{-3x}}{-3}(10x+7) + \frac{10e^{-3x}}{-9}\right] + C}$$

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$\int u'v = u\cdot v - \int u\cdot v'$: תזכורת

תרגיל 6

$$\Rightarrow \int \underset{u'}{e^{5x}} \underset{v}{\sin 7x}\,dx = \frac{e^{5x}}{5}\cdot \sin 7x - \frac{7}{5}\underbrace{\int \cos 7x\, e^{5x}\,dx}_{①}$$

$u = \frac{e^{5x}}{5}$ $\quad V = \sin 7x$

$u' = e^{5x}$ $\quad V' = 7\cos 7x$

$$\int \cos 7x\, e^{5x}\,dx = \cos 7x\cdot\frac{e^{5x}}{5} + \frac{7}{5}\underbrace{\int \sin 7x\cdot e^{5x}}_{②}$$

$u = \frac{e^{5x}}{5}$ $\quad v = \cos 7x$

$u' = e^{5x}$ $\quad v' = -7\sin 7x$

$$= \frac{e^{5x}}{5}\sin 7x - \frac{7}{5}\left[\frac{e^{5x}}{5}\cos 7x + \frac{7}{5}\int \sin 7x\cdot e^{5x}\,dx\right]$$

$$= \frac{e^{5x}}{5}\sin 7x - \frac{7}{5}\cdot\frac{e^{5x}}{5}\cos 7x - \frac{49}{25}\int \sin 7x\cdot e^{5x}\,dx$$

$$\frac{\left(1+\frac{49}{25}\right)\int e^{5x}\sin 7x}{1+\frac{49}{25}} = \boxed{\frac{\frac{e^{5x}}{5}\sin 7x - \frac{7}{5}\,\frac{e^{5x}}{5}\cos 7x}{\left(1+\frac{49}{25}\right)} + (}$$

$$\Rightarrow \int \frac{3x^2 - 28x + 7}{(x^2+4)(x-5)}\, dx$$

$$= \frac{Ax}{(x^2+4)} + \frac{B}{x^2+4} + \frac{C}{x-5}$$

$$\vdots$$

$$\Rightarrow \sin^5 7x \cos^9 7x \, dx = \int \frac{\sin^{\overset{4}{\cancel{5}}} 7x \; t^9 \, dt}{-7\cancel{\sin 7x}} = -\frac{1}{7}\int (1-t^2)^2 t^9$$

$t = \cos 7x$

$1dt = -7\sin 7x \, dx$

$\frac{dt}{-7\sin 7x} = dx$

$$= -\frac{1}{7}\int (1-2t^2+t^4)t^9 \, dt$$

$$= -\frac{1}{7}\int (t^2 - 2t^{11} + t^{13})\, dt = -\frac{1}{7}\left[\frac{t^{10}}{10} - \frac{2t^{12}}{12} + \frac{t^{14}}{14}\right] =$$

$$\boxed{= -\frac{1}{7}\left[\frac{\cos^{10} 7x}{10} - 2\,\frac{\cos^{12} 7x}{12} + \frac{\cos^{14} 7x}{14}\right] + C}$$

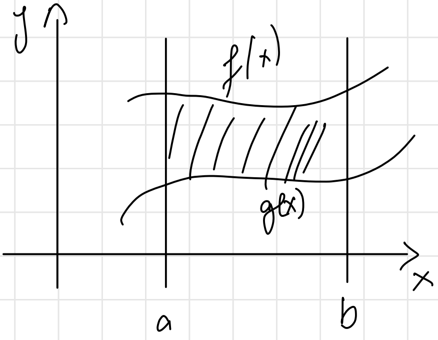
$$\Rightarrow \int \frac{dx}{\cos x + \sin x + 1} = \int \frac{2dt}{\frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} + 1}$$

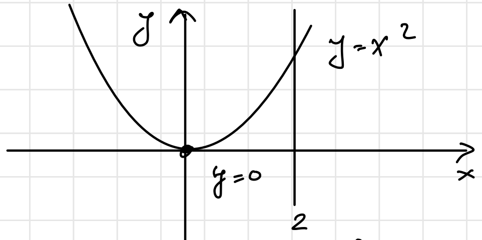
$$= \int \frac{\frac{2dt}{\cancel{1+t^2}}}{\frac{1-\cancel{t^2} + 2t + 1 + \cancel{t^2}}{\cancel{1+t^2}}} = \int \frac{2dt}{2t+2} = \int \frac{dt}{t+1}$$

$$= \ln(1+t) = \boxed{\ln\left(1+\tan\frac{x}{2}\right) + C}$$

1/5/22

הרצאה 7



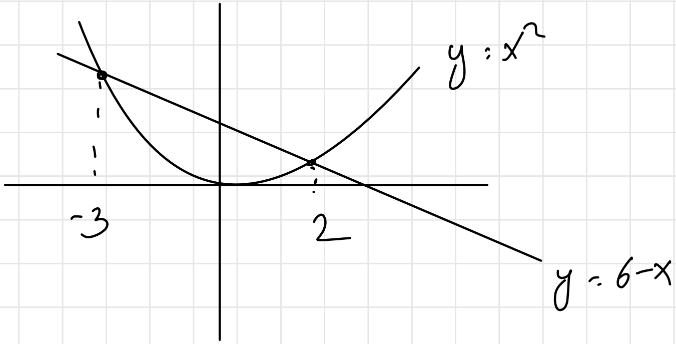


$$S=\int_a^b [f(x)-g(x)]\,dx$$

$$S=\int_0^2 (x^2-0)\,dx=\int_0^2 x^2\,dx$$

$$=\frac{x^3}{3}\Big|_0^2=\frac{8}{3}-0=\frac{8}{3}$$

עוד השלמות:



$x^2=6-x$ $\quad +3\ -2$ ✓

$0=x^2+x-6$

$0=(x+3)(x-2)$

$x_1=-3$

$x=2$

$$\int_{-3}^{2} x^2+x-6\,dx=\frac{x^3}{3}+\frac{x^2}{2}-6x\Big|_{-3}^{2}$$

-7.83

$$\left[\frac{8}{3}+\overset{2}{\frac{4}{2}}-12\right]$$

$$-\left[\frac{-3^3}{3}+\overset{\frac{27}{2}}{\frac{9}{2}}+18\right]$$

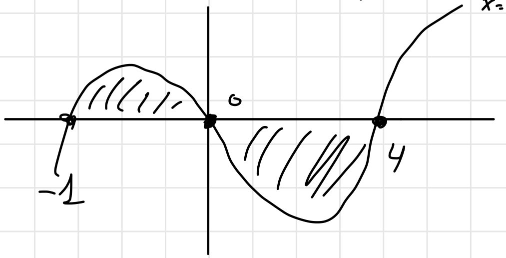
$= -20.83$

השטח הוא 20.83

$y = 0$

$y = x^3 - 3x^2 - 4x \quad x(x^2 - 3x - 4).$

$x = 0 \quad x = 4 \quad x = 1$



$f(x) = x^3 - 2x^2 + 9x + 5 \quad ; \quad g(x) = 2x^2 + 6x + 5$

$x^3 - 2x^2 + 9x + 5 - 2x^2 - 6x - 5 = 0$

$x^3 - 4x^2 + 3x = 0$

$x(x^2 - 4x + 3) = 0$

$x = 0 \quad x = 3 \quad x = 1$

$$\int_0^1 (f(x) - g(x))\,dx = \int_0^1 (x^3 - 4x^2 + 3x)\,dx = \left(\frac{x^4}{4} - \frac{4x^2}{2} + \frac{3x^2}{2}\right)\Bigg|_0^1 = \frac{5}{12}$$

$$\int_2^3 (x^3 - 4x^2 + 3x)\,dx = \left(\frac{x^4}{4} - \frac{4x^2}{2} + \frac{3x^2}{2}\right)\Bigg|_1^3 = \frac{-8}{3}$$

$$\text{nle } S = \frac{8}{3} + \frac{5}{12} = \frac{37}{12}$$

→ $f(x) = x^3 - 4x^2 + 13x - 7$ $\quad$ $g(x) = -x^3 + 6x^2 + 25x - 7$

$$f(x) = g(x)$$

$$2x^3 - 10x^2 - 12x = 0$$

$-6 \quad +1$

$$2x\left(x^2 - 5x - 6\right) = 0$$

$x = 0$ $\quad$ $x_{1,2} = 6, -1$

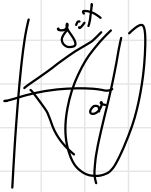
$$\int_{-1}^{0} f(x) - g(x)\,dx = \int_{-1}^{0} 2x^3 - 10x^2 - 12x\,dx = \frac{2x^4}{4} - \frac{10x^3}{3} - \frac{12x^2}{2}\Bigg|_{-1}^{0}$$

$$0 - -\frac{13}{6} = 2.166$$

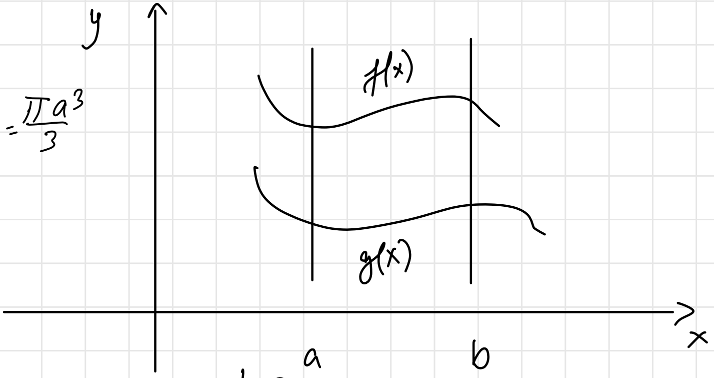
$$\int_{0}^{6} 2x^3 - 10x^2 - 12x\,dx = \frac{2x^4}{4} - \frac{10x^3}{3} - \frac{12x^2}{2}\Bigg|_{0}^{6} = -288$$

$$2.166 + 288 = 290.667$$

נפח גוף סיבוב סביב ציר ה- X



$V=\pi\int_0^a x^2dx=\pi\frac{x^3}{3}\Big|_0^a=\frac{\pi a^3}{3}$

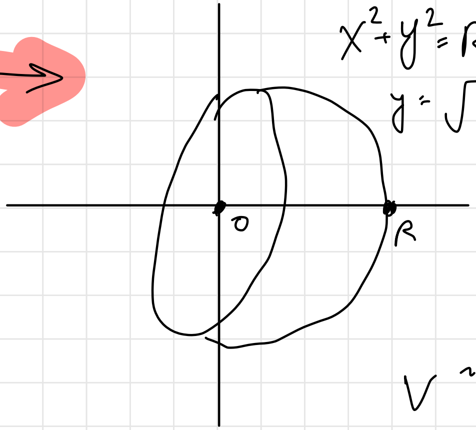


$$V=\pi\int_a^b\left(f^2(x)-g^2(x)\right)dx$$

→

$x^2+y^2=R^2$

$y=\sqrt{R^2-x^2}$



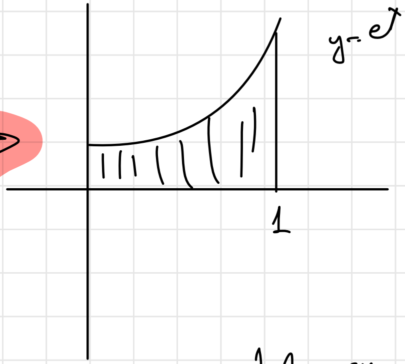
$$V=\pi\int_0^R\left(\sqrt{R^2-x^2}\right)dX$$

$$=\pi\int_0^R R^2-x^2\,dX=\pi\left(R^2X-\frac{X^3}{3}\right)\Bigg|_0^R=$$

$$\pi\left(R^2\cdot R-\frac{R^3}{3}\right)=\frac{2\pi R^3}{3}$$

נפח חצי כדור

$\frac{4\pi R^3}{3}$ נפח כדור

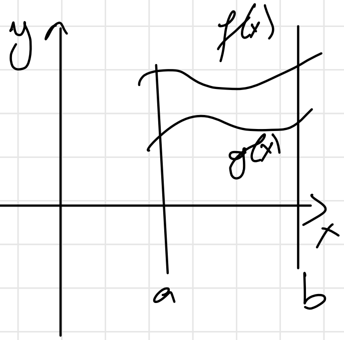


$$V = \pi \int_0^1 \left((e^x)^2 - 0^2 \right) dx$$

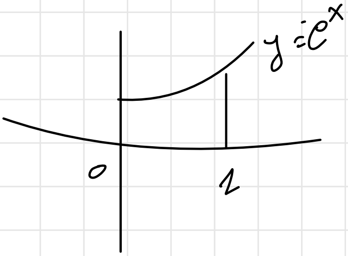
$$= \pi \int_0^1 e^{2x} dx = \pi \frac{e^{2x}}{2} \Big|_0^2$$

$$\pi \left(\frac{e^2}{2} - \frac{e^0}{2} \right) = \boxed{\frac{\pi}{2} \left(e^2 - 1 \right)}$$

נפח גוף סיבוב סביב ציר ה-y



$$V = 2\pi \int_0^b x \left(f(x) - g(x) \right) dx$$



$$V = 2\pi \int_0^1 x \left(e^x - 0 \right) dx$$

$$= \int_0^1 \underset{u'}{x} \underset{v}{e^x} dx = e^x \cdot x - \int_0^1 e^x dx$$

$$\int \underset{v}{x}\, \underset{u'}{e^x}\, dx \qquad e^x \cdot x - e^x \Big|_0^1$$

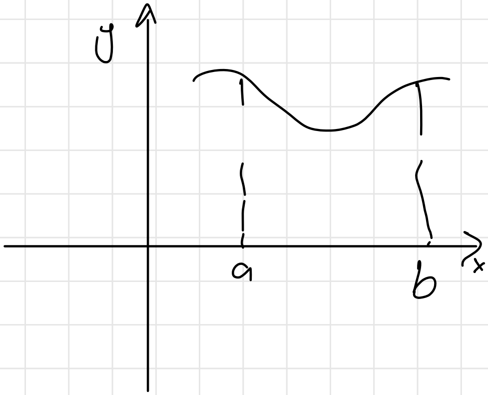
$$u = e^x \quad v = x$$

$$u' = e^x \quad v' = 1$$

$$[e - e] - [0 - 1] = 1$$

$$V = 2\pi$$

אורך קטע



$$L = \int_a^b \sqrt{2 + (f'(x))^2}\, dx$$

$$y = \frac{1}{3}(x^2+2)^{\frac{3}{2}} \qquad 0 \leq x \leq 3$$

$$y' = \frac{1}{3} \cdot \frac{3}{2}(x^2+1)^{\frac{1}{2}} \cdot 2x = (x^2+2)^{\frac{1}{2}} \cdot x$$

$$1 + (y')^2 = 1 + (x^2+2) \cdot x^2 = x^4 + 2x^2 + 1 = (x^2+2)^2$$

$$L = \int_0^3 \sqrt{1 + (y')^2}\, dx = \int_0^3 \sqrt{(x^2+1)^2}\, dx = \int_0^3 (x^2+1)\, dx$$

$$= \left(\frac{x^3}{3} + x\right)\Bigg|_0^3 = \frac{3^3}{3} + 3 - 0 = \underline{\underline{12}}$$

$2ab = \frac{-1}{2}$ [illegible]

$(a-b)^2 + 1 = (a+b)^2$

$(a-b)^2 \Rightarrow (a+b)^2$

$1 \leq x \leq 3$

→ $y = \frac{x^4}{8} + \frac{1}{4x^2} = \frac{x^4}{8} + \frac{x^{-2}}{4}$

$$y' = \frac{4x^3}{8} - \frac{2x^{-3}}{4} = \frac{x^3}{2} - \frac{x^{-3}}{2}$$

$$1 + (y')^2 = 1 + \left(\frac{x^3}{2} - \frac{x^{-3}}{2}\right)^2 = 1 + \left(\frac{x^3}{2}\right)^2 \underbrace{- 2 \cdot \underbrace{\frac{x^3}{2} \cdot \frac{x^{-3}}{2}}_{\frac{1}{4}}}_{-\frac{1}{2}} + \left(\frac{x^{-3}}{2}\right)^2$$

$$-\frac{1}{2} = \frac{1}{2}$$

$$= \left(\frac{x^3}{2} + \frac{x^{-3}}{2}\right)^2$$

$$L = \int_1^3 1 + (f'(x))^2 \, dx = \int_1^3 \sqrt{\left(\frac{x^3}{2} + \frac{x^{-3}}{2}\right)^2} \, dx = \int_1^3 \left(\frac{x^3}{2} + \frac{x^{-3}}{2}\right) dx$$

$$= \left(\frac{x^4}{2 \cdot 4} + \frac{x^{-2}}{2 \cdot -2}\right)\Bigg|_1^3 = 10.22$$

→ $y = \ln(\sin x)$ $\left(\frac{\pi}{3} \leq x \leq \frac{\pi}{2}\right)$

$$y' = \frac{1}{\sin x} \cdot \cos x = \cot x$$

$$1 + (y')^2 = 1 + \cot^2 x = \frac{1}{\sin^2 x}$$

$$L = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 1 + (f'(x))^2 = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{\frac{1}{\sin^2 x}}\, dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\sin x}\, dx$$

$$= \ln\left(\tan\frac{x}{2}\right)\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \ln\Big(\underset{\overset{\shortparallel}{1}}{\tan\frac{\pi}{4}}\Big) - \ln\left(\tan\frac{\pi}{6}\right) = \overset{0}{\cancel{\ln(1)}} - \ln\left(\frac{1}{\sqrt{3}}\right) =$$

$$\boxed{\ln\left(\frac{1}{\sqrt{3}}\right)}$$

$$\int \frac{1}{\sin x}\, dx = \boxed{\int \frac{1}{\frac{\cancel{2}t}{\cancel{t+1^2}}} \cdot \frac{\cancel{2}dt}{\cancel{1+t^2}} = \int \frac{1}{t}\, dt = \ln t = \ln\left(\tan\frac{x}{2}\right) + C}$$

$t = \tan\frac{x}{2}$

$\sin x = \frac{2t}{1+t^2}$

$dx = \frac{2dt}{1+t^2}$

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תרגול 7

①

$y=\frac{e^{2x}+e^{-2x}}{4}$ $0\leq x\leq 1$

$$y'=\frac{e^{2x}\cdot 2-2e^{-2x}}{4}=\frac{e^{2x}}{2}-\frac{e^{-2x}}{2}$$

$$1+(y')^2=1+\left(\frac{e^{2x}}{2}-\frac{e^{-2x}}{2}\right)^2=\left(\frac{e^{2x}}{2}+\frac{e^{-2x}}{2}\right)^2$$

$$L=\int_0^1\sqrt{\left(\frac{e^{2x}}{2}+\frac{e^{-2x}}{2}\right)^2}\,dx=\int_0^1\left(\frac{e^{2x}}{2}+\frac{e^{-2x}}{2}\right)dx=\left(\frac{e^{2x}}{2}-\frac{e^{-2x}}{2}\right)\Bigg|_0^1$$

$$\boxed{\approx 1.81}$$

②

$1\leq x\leq 2$ $y=x^2-\frac{\ln x}{8}$

$$y'=2x-\frac{1}{8x}=2x-\frac{x^{-1}}{8}$$

$$1+(y')^2=1+\left(2x-\frac{x^{-1}}{8}\right)^2=1+4x^2-2\cdot 2x\cdot\frac{x^{-1}}{8}+\left(\frac{x^{-1}}{8}\right)^2=\left(2x+\frac{x^{-1}}{8}\right)^2$$

$$L=\int_1^2\left(2x+\frac{x^{-1}}{8}\right)dx=\frac{2x^2}{2}+\frac{\ln x}{8}\Bigg|_1^2$$

$$=4+\frac{\ln 2}{8}-\left(1-\frac{\ln 1}{8}\right)=3+\frac{\ln 2}{8}$$

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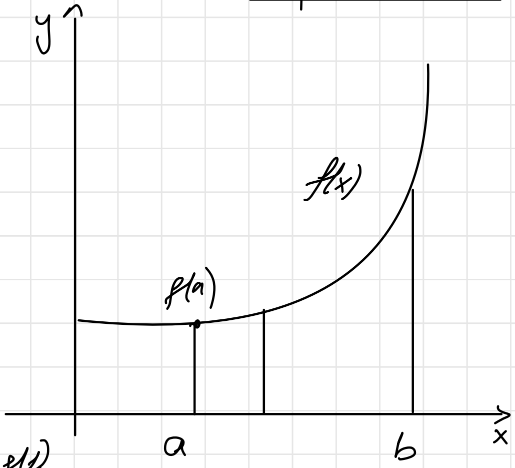
הרצאה 8

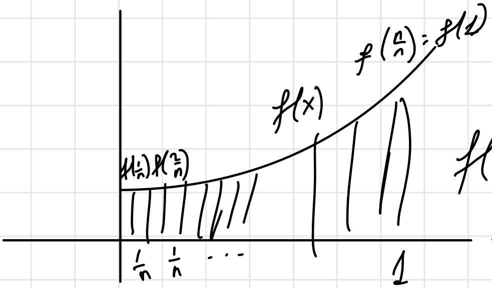
סכומי רימן

$$\int_a^b f(x)\,dx$$

$f(x)$ אינטגרבילית בקטע $[a,b]$

אם קיים האינטגרל בקטע והתוצאה סופית





$$f\left(\frac{1}{n}\right)\cdot\frac{1}{n}+f\left(\frac{2}{n}\right)\cdot\frac{1}{n}+\dots+f\left(\frac{n}{n}\right)\cdot\frac{1}{n}$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[f\left(\frac{1}{n}\right)+\dots+f\left(\frac{n}{n}\right)\right]$$

$$=\int_0^1 f(x)\,dx$$

$$\lim_{n\to\infty}\frac{1}{n}\left[f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+\dots+f\left(\frac{n}{n}\right)\right]=\int_0^1 f(x)\,dx$$

$$\lim_{n\to\infty}\frac{1}{n}\left[\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\left(\frac{3}{n}\right)^2+\dots+\left(\frac{n}{n}\right)^2\right]=\int_0^1 x^2\,dx=\frac{x^3}{3}\Big|_0^1=\frac{1}{3}$$

$$f(x)=x^2$$

$$\lim_{n \to \infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \dots + \frac{n}{n^2} \right) = \lim_{n \to \infty} \frac{1}{n} \left[\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots + \frac{n}{n} \right]$$

$$= \int_0^1 x \, dx = \frac{x^2}{2} \Big|_0^1 = \frac{1}{2}$$

$$= \lim_{n \to \infty} \frac{1+2+3+ \dots + n}{n^2} = \lim_{n \to \infty} \frac{n(n+1)}{2n^2} = \frac{1}{2}$$

$$\lim_{n \to \infty} \left(\frac{3n}{(n+1)^2} + \frac{3n}{(n+2)^2} + \frac{3n}{(n+3)^2} + \dots + \frac{3n}{(n+n)^2} \right) =$$

$$3 \cdot \lim_{n \to \infty} \frac{1}{n} \left[\frac{n^2}{(n+1)^2} + \frac{n^2}{(n+2)^2} + \frac{n^2}{(n+3)^2} + \dots + \frac{n^2}{(n+n)^2} \right]$$

$$= 3 \lim_{n \to \infty} \frac{1}{n} \left[\left(\frac{n}{n+1} \right)^2 + \left(\frac{n}{n+2} \right)^2 + \dots + \left(\frac{n}{n+n} \right)^2 \right]$$

$$= 3 \lim_{n \to \infty} \frac{1}{n} \left[\left(\frac{1}{1+\frac{1}{n}} \right)^2 + \left(\frac{1}{1+\frac{2}{n}} \right)^2 + \dots + \left(\frac{1}{1+1} \right)^2 \right]$$

$$= 3 \lim_{n \to \infty} \frac{1}{n} \left[\frac{1}{(1+\frac{1}{n})^2} + \frac{1}{(1+\frac{2}{n})^2} + \dots + \frac{1}{(1+1)^2} \right]$$

$$= 3 \cdot \int_0^1 \frac{1}{(1+x)^2} dx = 3 \cdot \int_0^1 (1+x)^{-2} dx = 3 \frac{(1+x)^{-1}}{-1} \Big|_0^1 = 3 \cdot \frac{-1}{1+x} \Big|_0^1 = \frac{3}{2}$$

→ $$\lim_{n\to\infty}\left(\frac{n}{(n^2+1^2)}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\dots+\frac{n}{n^2+n^2}\right)=\lim_{n\to\infty}\frac{1}{n}\left[\frac{n^2}{n^2+1^2}+\frac{n^2}{n^2+2^2}+\dots\frac{n^2}{n^2+n^2}\right]$$

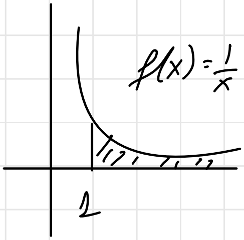
$$=\lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{1+(\frac{1}{n})^2}+\frac{1}{1+(\frac{2}{n})^2}+\frac{1}{1+(\frac{3}{n})^2}+\dots+\frac{1}{1+(\frac{n}{n})^2}\right]=\int_0^1\frac{1}{1+x^2}dx$$

$$=\arctan x\Big|_0^1=\underset{\frac{\pi}{4}}{\arctan 1}-\arctan 0=\frac{\pi}{4}$$

→ $$\lim_{n\to\infty}\frac{1}{n}\left[\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\sin\frac{3\pi}{n}+\dots+\sin\frac{n\pi}{n}\right]=\int_0^1\sin(\pi x)\,dx=\frac{-\cos\pi x}{\pi}\Big|_0^1$$

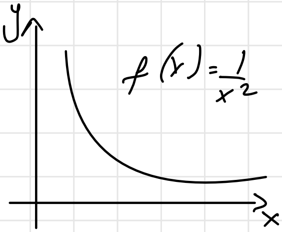
$$=-\frac{\cos^{-1}(1\cdot\pi)}{\pi}-\left(\frac{-\overset{1}{\cancel{\cos}}(0)}{\pi}\right)=\frac{1}{\pi}+\frac{1}{\pi}=\frac{2}{\pi}$$

אנטגרלים לא אמיתיים



$$\int_1^\infty\frac{1}{x}dx=\lim_{t\to\infty}\int_1^t\frac{1}{x}dx=\lim_{t\to\infty}\ln x\Big|_1^t$$

$$=\lim_{t\to\infty}(\ln t-\overset{0}{\cancel{\ln 1}})=\infty$$



$$\int_1^\infty\frac{1}{x^2}dx=\lim_{t\to\infty}\int_1^t x^{-2}dx=\lim_{t\to\infty}\frac{-1}{x}\Big|_1^t$$

$$=\lim_{t\to\infty}\left[\overset{0}{\cancel{\frac{-1}{t}}}-\left(\frac{-1}{1}\right)\right]=1$$

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}\Big|_0^1 = 2\cdot 1 - 0 = 2$$

$$\int_0^1 \frac{1}{x}\,dx = \ln x\Big|_0^1 = \cancel{\overset{0}{\ln 1}} - \cancel{\overset{-\infty}{\ln 0}} = \infty$$

→ $\displaystyle\int_1^\infty \frac{1}{x}\,dx$ לא מתכנס

→ $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx =$ מתכנס

$$\int_1^\infty \frac{1}{x^\alpha}\,dx = \int_1^\infty x^{-\alpha}\,dx = \begin{cases} \dfrac{x^{-\alpha+1}}{-\alpha+1}\Big|_1^\infty & \alpha \neq 1 \\ \ln x\Big|_1^\infty & \alpha = 1 \end{cases} =$$

$= \dfrac{x^{1-\alpha}}{1-\alpha}\Big|_1^\infty$ $\quad \alpha > 1$ מתכנס

$\ln x\Big|_1^\infty$ $\quad \alpha = 1$ מתבדר

$\dfrac{x^{1-\alpha}}{1-\alpha}\Big|_1^\infty$ $\quad \alpha < 1$ מתבדר

$\int_1^\infty \frac{1}{x^\alpha} dx =$ מתכנס $x>1$
מתבדר $\alpha \le 1$

$\frac{1}{3}$ מתכנס
גודל אינסופי – מתבדר

מבחן השוואה ראשון :

נתון : $f(x) \ge g(x) \ge 0$

מכאן : $\int_1^\infty g(x)\,dx \le \int_1^\infty f(x)\,dx$

אם $\int_1^\infty f(x)\,dx$ מתכנס אזי $\int_1^\infty g(x)\,dx$ מתכנס.

אם $\int_1^\infty g(x)\,dx$ מתבדר אזי $\int_1^\infty f(x)\,dx$ מתבדר.

→ $\int_1^\infty \frac{1}{x^3+7}\,dx \le \underbrace{\int_1^\infty \frac{1}{x^3}\,dx}$

מתכנס $\Leftarrow$ מתכנס

→ $\underbrace{\int_2^\infty \frac{1}{x}}_{מתבדר} \le \int_2^\infty \frac{1}{\ln x}\,dx$

$2 \le x$

מתבדר $\Rightarrow$ מתבדר

$\ln x \le x$

$\frac{1}{\ln x} \ge \frac{1}{x}$

מבחן השוואה הגבולי :

נתון : $f(x), g(x) \geq 0$

וכמו כן $\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$

$(0<L<\infty)$

אזי $\int_1^\infty f(x)\,dx$ ו $\int_1^\infty g(x)\,dx$ מתכנסים יחדיו או מתבדרים יחדיו.

$$\int_1^\infty \frac{2x^2+3}{5x^4+7x^2-9}\,dx$$

מתכנס

$$\lim_{x\to\infty} \frac{\frac{2x^2+3}{5x^4+7x^2-9}}{\frac{1}{x^2}} = \frac{2x^4+3x^2}{5x^4+7x^2-9} = \frac{2}{5}$$

מתכנס

8/5/22

תרגול 8

$$x^{\frac{2}{3}}+y^{\frac{2}{3}}=1$$

$$y^{\frac{2}{3}}=1-x^{\frac{2}{3}}$$

$$y=\left(1-y^{\frac{2}{3}}\right)^{\frac{3}{2}}$$

$$y'=\cancel{\frac{3}{2}}\left(1-y^{\frac{2}{3}}\right)^{\frac{1}{2}}\cdot\cancel{\frac{2}{3}}\,x^{-\frac{1}{3}}=\left(1-x^{\frac{2}{3}}\right)^{\frac{1}{2}}x^{-\frac{1}{3}}(-1)$$

$$\lim_{n\to\infty}\frac{1^7+2^7+3^7+\cdots+n^7}{n^8}=\lim_{n\to\infty}\frac{1}{n}\left[\frac{1^7+2^7+\cdots+n^7}{n^7}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\left(\frac{1}{n}\right)^7+\left(\frac{2}{n}\right)^7+\cdots+\left(\frac{n}{n}\right)^7\right]=\int_0^1 x^7\,dx=\frac{x^8}{8}\Big|_0^1=\frac{1}{8}$$

$$\lim_{n\to\infty}\frac{\pi}{2n}\left[1+\cos\frac{\pi}{2n}+\cos\frac{2\pi}{2n}+\cos\frac{3\pi}{2n}+\cdots+\cos\frac{(n-1)\pi}{2n}\right.$$

✓

$$=\frac{\pi}{2}\int_0^1\cos\left(\frac{\pi x}{2}\right)dx=\frac{\cancel{\frac{\pi}{2}}\sin\left(\frac{\pi x}{2}\right)}{\cancel{\frac{\pi}{2}}}\Bigg|_0^1=\cancel{\sin\frac{\pi}{2}}^{\,1}-\cancel{\sin 0}^{\,0}=1$$

→ $$\lim_{n\to\infty} \frac{1}{\sqrt{9n^2-1^2}} + \frac{1}{\sqrt{9n^2-2^2}} + \cdots + \frac{1}{\sqrt{9n^2-n^2}}$$

$$= \lim_{n\to\infty} \left[\frac{1}{3n} \frac{1}{\sqrt{1-\left(\frac{1}{3n}\right)^2}} + \frac{1}{3n}\cdot\frac{1}{\sqrt{1-\left(\frac{2}{3n}\right)^2}} + \cdots + \frac{1}{3n}\cdot\frac{1}{\sqrt{1-\left(\frac{n}{3n}\right)^2}} \right)$$

$$= \frac{1}{3}\lim \frac{1}{n}\left[\frac{1}{\sqrt{1-\left(\frac{1}{3n}\right)^2}} + \cdots + \frac{1}{\sqrt{1-\left(\frac{n}{3n}\right)^2}}\right] = \frac{1}{3}\int_0^1 \frac{1}{\sqrt{1-\left(\frac{x}{3}\right)^2}}\,dx$$

$$= \arcsin\frac{1}{3} - \cancelto{0}{\arcsin 0} = \arcsin\frac{1}{3} \qquad ar$$

→ $$\lim_{n\to\infty} 7n\left(\frac{1}{25n^2+1^2} + \frac{1}{25n^2+2^2}+\cdots+\frac{1}{25n^2+n^2}\right)$$

$$7\lim\frac{1}{n}\left(\frac{n^2}{25n^2+1^2}+\frac{n^2}{25n^2+2^2}+\cdots+\frac{n^2}{25n^2+n^2}\right)$$

$$= 7\lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{25+\left(\frac{1}{n}\right)^2}+\frac{1}{25+\left(\frac{2}{n}\right)^2}+\cdots+\frac{1}{25+\left(\frac{n}{n}\right)^2}\right]$$

$$= 7\int_0^1\frac{1}{25+x^2}\,dx = 7\cdot\frac{1}{5}\arctan\frac{x}{5}\Big|_0^1 =$$

$$= \frac{7}{5}\left[\arctan\frac{1}{5} - \arctan 0\right] = \frac{7}{5}\arctan\frac{1}{3}$$

$$\int_0^1 \frac{1}{1+\left(\frac{x}{5}\right)^2}\,dx = \frac{\arctan\frac{x}{5}}{\frac{1}{5}}$$

$$\int_1^\infty \frac{1}{x^\alpha}\,dx = \begin{cases} \text{מתכנס} & \alpha>1 \\ \text{מתבדר} & \alpha\le 1 \end{cases}$$

→ $$\int_2^\infty \frac{7x^3-5x^2+9}{5x^4+13x}\,dx$$

מתבדר

$$\lim_{x\to\infty} \frac{\frac{7x^3-5x^2+9}{5x^4+13x}}{\frac{1}{x}}\,dx = \lim_{x\to\infty} \frac{7x^4-5x^3+9x}{5x^4+13x} = \frac{7}{5}$$

מתבדר

סופי לכן מבחן השוואה בגבול

→ $$\int_2^\infty \frac{1}{x\ln x}\,dx = \int \frac{\frac{1}{x}}{\ln x}\,dx = \ln(\ln x)+C$$

$$= \ln(\ln x)\Big|_2^\infty$$

$$\ln(\underset{\infty}{\ln\infty}) - \underset{\text{סופי}}{\ln(\ln 2)} \Rightarrow \text{מתבדר}$$

أنا ساكن

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הרצאה 9

$t=\ln x \rightarrow$

$dt=\frac{1}{x}dx$

$x\,dt=dx$

$$\int_5^\infty \frac{1}{x\ln^2 x}dx = \int_5^\infty \frac{\frac{1}{\cancel{x}}}{t^2}\cancel{x}\cdot dt = \int_{5'}^\infty t^{-2}\,dt = \frac{t^{-1}}{-1} = \frac{-1}{\ln(x)}$$

$$= \boxed{\frac{-1}{\boxed{\ln(\infty)}\to\infty}}\to 0 - \left(-\frac{1}{\ln(5)}\right) \Rightarrow \frac{1}{\ln(5)}.$$ $\Rightarrow$ מתכנס

מבחן דריכלה

נתון $f(x)$ אינטגרבילית לפי x המקיימת $\left|\int_a^b f(x)dx\right| \le M$

וכמו כן נתון $g(x)$ מונוטונית יורדת המקיימת $\lim g(x)=0$

אזי האינטגרל $\int_a^\infty f(x)g(x)dx$ מתכנס.

$$\Rightarrow \int_1^\infty \frac{\sin x}{x}dx = \int_1^\infty \sin x \cdot \frac{1}{x}dx$$

$$\left|\int_a^b \sin x\,dx\right| = \left|-\cos x\Big|_a^b\right| = \left|-\cos b-(-\cos a)\right| \le 2$$

$$g(x)=\frac{1}{x} \underset{x\to\infty}{\Rightarrow} 0$$

$$g'(x)=\frac{-1}{x^2}<0$$ לכן! מונוטונית

יורדת. לכן! מקיימת את תנאי מבחן דריכלה לכן! מתכנס.

$$\underbrace{\int f(x)\,dx}_{\text{מתכנס}} \pm \underbrace{\int g(x)\,dx}_{\text{מתרנס}} = \text{מתכנס}$$

מתרנס מתבדר

$$\int_1^\infty \frac{\cos^2 x}{x}\,dx = \int_1^\infty \frac{1}{x}\cdot\cos^2 x\,dx = \int_1^\infty \frac{1+\cos 2x}{2x}\,dx$$

מתכנס

$$= \underbrace{\int_1^\infty \frac{1}{2x}\,dx}_{\text{מתבדר}} + \underbrace{\int_1^\infty \frac{\cos 2x}{2x}\,dx}_{} = \text{מתבדר}$$

$$\int_a^b \cos 2x\,dx = \frac{\sin 2x}{2}\Big|_a^b = \left|\frac{\sin 2b}{2} - \frac{\sin 2a}{2}\right| \leq 1$$

$$g(x) = \frac{1}{x} \to 0$$

$$g'(x) = \frac{-1}{x^2} \leq 0$$

מונוטונית יורדת לכן לפי דיריכלה

מתכנס

$$\int_1^\infty x^4 e^{-x}\cos x\,dx = \int_1^4 x^4 e^{-x}\cos x\,dx + \int_4^\infty x^4 e^{-x}\cos x\,dx$$

שואף אינסוף

חסום

$$\rightarrow \left(\frac{x^4}{e^x}\right)' = \frac{4x^3e^x - x^4e^x}{e^{2x}} = \frac{4x^3e^x - x^4e^x}{e^{2x}}$$

$$= \frac{x^3 e^x (4-x)}{e^{2x}} \rightarrow$$ מונוטונית החל מ- $x=4$ → מקיימת בררי∫ב → מתכנס

→ $$\int_1^\infty \frac{\sin^2 x}{x^3}\,dx \leqslant \int_1^\infty \frac{1}{x^3}\,dx$$

מתכנס ⇍ מתכנס

טורים

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \cdots + a_n + \cdots$$

$\int_1^\infty \frac{1}{x}\,dx$ מתבדר $\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} + \cdots$

$\int_1^\infty \frac{1}{x^2}\,dx$ מתכנס $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} + \cdots$

מתכנס $\sum_{1=n}^{\infty} \frac{1}{3^n} = \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^n}$

מתבדר $\sum_{n=1}^{\infty} (2n+3) = 5 + 7 + 9 + \cdots$

$$\sum (-1)^n = -1 + 1 + -1 + \cdots$$

טור טלסקופי' $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right) = \frac{1}{1} \cancel{-\frac{1}{2}} \cancel{+\frac{1}{2}} \cancel{-\frac{1}{3}} \cancel{+} \cancel{\ldots} \cancel{+\frac{1}{n}} - \frac{1}{n+1} = 1 - \frac{1}{n+1} \xrightarrow[n\to\infty]{} 1$$

$$\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right) = \sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right) = \sum_{n=1}^{\infty} \left[\ln(n+1) - \ln(n)\right] = \cancel{\ln 2} - \overset{0}{\cancel{\ln 1}} + \cancel{\ln 3} - \cancel{\ln 2} + \ln 4 - \cancel{\ln 3}$$

$$\ldots + \ln(n+1) - \cancel{\ln(n)}$$

$$= \ln(n+1) \xrightarrow[n\to\infty]{} \infty$$

ולכן מתבדר!

תנאי הכרחי להתכנסות טור $\sum a_n$ הוא e $\lim_{n\to\infty} a_n = 0$

22/5/22

הרצאה 10

טורים חיוביים:

① מבחן ההשוואה ראשון

נתון: $b_n \le a_n$ החל ממקום מסוים.

אזי: אם $\sum a_n$ מתכנס ⇐ $\sum b_n$ מתכנס

אם $\sum b_n$ מתבדר ⇐ $\sum a_n$ מתבדר

$$\sum \frac{1}{n^5+17} \le \sum \frac{1}{n^5}$$

מתכנס ⇐ מתכנס

$$\sum \frac{1}{n} \le \sum_{n=2}^{\infty} \frac{1}{\ln n}$$

מתבדר → מתבדר

$$\ln n < n$$

$$\frac{1}{\ln n} > \frac{1}{n}$$

② מבחן ההשוואה הגבולי

נתונים $a_n, b_n \ge 0$ החל ממקום מסוים.

כמו כן נתון: $\lim_{n\to\infty} \frac{a_n}{b_n} = L$

$L \ne 0$ סופי

אזי $\sum a_n$ ו $\sum b_n$ מתכנסים או מתבדרים יחדיו.

מתכנס $\rightarrow \sum \frac{3n^3+7n^2-2}{7n^5+4n^2-3} = ?$

$$\lim_{n\to\infty} \frac{\frac{3n^3+7n^2-2}{7n^5+4n^2-3}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{3n^5+7n^4-2n^2}{7n^5+4n^2-3} = \frac{3}{7}$$

מתכנס $\frac{1}{n^2}$

תנאי הכרחי להתכנסות הטור $\sum a_n$ הוא $\lim_{n\to\infty} a_n = 0$

וואלה

③ מבחן המנה (מבחן דלמבר)

נתון: $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$

אם $L<1 \Leftarrow \sum a_n$ מתכנס

אם $L>1 \Leftarrow \sum a_n$ מתבדר

אם $L=1 \Leftarrow \sum a_n$ לא ידוע

$\rightarrow \sum \frac{1}{n}$

$$\frac{a_{n+1}}{a_n} = \frac{1}{n+1} \cdot \frac{n}{1} = \frac{n}{n+1} \xrightarrow[n\to\infty]{} 1$$

לא ידוע!

$$\rightarrow \sum \frac{1}{3^n}$$

$$\frac{a_{n+1}}{a_n} = \frac{1}{3^{n+1}} \cdot \frac{3^n}{1} = \frac{\cancel{3^n}}{3 \cdot \cancel{3^n}} = \frac{1}{3} < 1$$

לפי דלמבר מתכנס

$$\rightarrow a_n = \frac{e^{3n+5}}{7n-2} \qquad a_{n+1} = \frac{e^{3(n+1)+5}}{7(n+1)-2} = \frac{e^{3n+8}}{7n+5}$$

$$\sum \frac{(n!)^2}{2n!} = \frac{(n+1)!^2}{(2n+2)!} \cdot \frac{2n!}{(n!)^2} = \frac{\cancel{n!} \cdot (n+1) \cdot \cancel{n!}\,\cancel{(n+1)} \cdot \cancel{2n!}}{\cancel{2n!} \cdot 2n+1 \cdot \underset{2\cancel{(n+1)}}{2n+2} \cdot \cancel{n!} \cdot \cancel{n!}} = \frac{n+1}{4n+2}$$

$$= \frac{1}{4} < 1$$

מתכנס.

$$\rightarrow \sum \frac{2^n \cdot n!}{n^n} = \frac{2^{n+1} \cdot n! \cdot n+1}{(n+1)^{n+1}} \cdot \frac{n^n}{2^n \cdot n!} = \frac{2 \cdot \cancel{2^n} \cdot \cancel{n!}\,\cancel{(n+1)} \cdot n^n}{(n+1)^n \cdot \cancel{(n+1)} \cdot \cancel{2^n} \cdot \cancel{n!}}$$

$$= 2\left(\frac{n}{n+1}\right)^n \rightarrow 2e^{-1} = \frac{2}{e} < 1$$

$$\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = e^L$$

$$L = \left(\frac{n}{n+1} - 1\right)^n = \lim_{n\to\infty}\left(\frac{n+n-1}{n+1}\right)^n = \lim_{n\to\infty} \frac{-n}{n+1} = 1$$

$$\sum_{n=1}^{\infty} \sin\frac{1}{n}$$

לכן מתבדר

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

מתבדר

$$\lim_{n\to\infty}\frac{\sin\frac{1}{n}}{\frac{1}{n}}=1$$

$$\sum \sin^2\frac{1}{n} = \lim_{n\to\infty}\frac{\sin^2\frac{1}{n}}{\frac{1}{n^2}} = \lim_{n\to\infty}\left(\frac{\sin\frac{1}{n}}{\frac{1}{n}}\right)^2=1$$

מתכנס

מתכנס

מבחן השורש

נתון $\lim_{n\to\infty}\sqrt[n]{a_n}=L$

אם $\sum a_n \Leftarrow L<1$ מתכנס

אם $\sum a_n \Leftarrow L>1$ מתבדר

אם $\sum a_n \Leftarrow L=1$ לא ידוע

$$\lim \frac{a_{n+1}}{a_n} = \lim \sqrt[n]{a_n}$$

$$\sum \frac{1}{3^n}$$

$$\sqrt[n]{a_n}=\sqrt[n]{\frac{1}{3}} \qquad \frac{1}{3}<1$$

מתכנס

$$\rightarrow \sum \frac{1}{\left(\frac{n+1}{n+3}\right)^{n(n+1)}}$$

$$\sqrt[n]{a_n} = \sqrt[n]{\frac{1}{\left(\frac{n+1}{n+3}\right)^{n(n+1)}}} = \frac{1}{\left(\frac{n+1}{n+3}\right)^{n+1}} =$$

$$\lim_{n\to\infty} \left(\frac{n+1}{n+3}\right)^{n+1} = e^{L}$$

$$L = \lim_{n\to\infty} \left(\frac{n+1}{n+3} - 1\right)(n+1) = \lim_{n\to\infty} \left(\frac{\cancel{n}+1 - \cancel{n} - 3}{n+3}\right)^{n+1}$$

$$= \lim_{n\to\infty} \frac{-2n-2}{n+3} = -2$$

$$\sum \left(\frac{n}{n+1}\right)^{2n} = \frac{1}{e^2} \neq 0$$ מתבדר.

מבחן האינטגרל

נתון $\preceq f(x)$ לכל x

אזי $\int_1^\infty f(x)$! $\sum_{n=1}^\infty f(n)$

מתכנסים או מתבדרים יחדיו.

$$\sum_{n=5}^\infty \frac{1}{n\ln n} = \int_{\ln 5}^\infty \frac{1}{\cancel{x}t}\cancel{x}dt = \int_{\ln 5}^\infty \frac{1}{t}dt$$

מתבדר

$t = \ln x$

$x = 5 \Rightarrow t = \ln 5$

$x = \infty \Rightarrow \ln\infty = \infty$

$\cancel{1}dt = \frac{1}{x}dx$

$dt = xdt$

$\sum \frac{1}{n\ln n}$ מתבדר

לצאות! לפי מבחן האינטגרל

$$\sum \frac{1!+2!+3!+\dots+n!}{(2n)!} \le \sum \frac{n!+n!+n!+\dots+n!}{2n!} = \sum\frac{n\cdot n!}{2n!}$$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)(n+1)!}{(2n+2)!}\cdot\frac{2n!}{n\,n!} = \frac{n+1}{4n^2+2n} \Rightarrow 0$$

לכן מתכנס

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